SRI CHANDRASEKHARENDRA SARASWATHI VISWA MAHAVIDYALAYA

(University established under section 3 of UGC Act 1956) (Accredited With 'A' Grade by NAAC)

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

Course Material on

ANTENNAS AND PROPAGATION

V SEMESTER (PROGRAM ELECTIVE)

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PEC1 ANTENNAS AND PROPAGATION V SEMESTER

PRE-REQUISITE:

Basic knowledge of Electromagnetic Fields and Wave guides

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OBJECTIVES:

- To give insight of radiation phenomena.
- To give thorough understanding of the radiation characteristics of different types of antenna
- To create awareness about different types of propagation of radio waves at different Frequencies

UNIT I FUNDAMENTALS OF RADIATION (9 Hrs)

Definition of antenna parameters – Gain, Directivity, Effective aperture, Radiation Resistance, Band width, Beam width, Input Impedance, Matching Baluns, Polarization mismatch, Antenna noise temperature, Radiation from oscillating dipole, Half wave dipole. Folded dipole

UNIT II ANTENNA ARRAYS (9 Hrs)

N element linear array, Pattern multiplication, Broadside and End fire array – Concept of Phased arrays, Adaptive array, Basic principle of antenna Synthesis-Binomial array, Yagi Arrays.

UNIT III APERTURE AND SLOT ANTENNAS (9 Hrs)

Radiation from rectangular apertures, Uniform and Tapered aperture, Horn antenna, Reflector antenna, Aperture blockage, Feeding structures, Slot antennas, Micro strip antennas – Radiation mechanism – Application, Numerical tool for antenna analysis

UNIT IV SPECIAL ANTENNAS (9 Hrs)

Principle of frequency, independent antennas –Spiral antenna, helical antenna, Log periodic, Modern antennas- Reconfigurable antenna, Active antenna, Dielectric antennas, Electronic band gap structure and applications, Antenna Measurements-Test Ranges, Measurement of Gain, Radiation pattern, Polarization, VSWR

UNIT V PROPAGATION OF RADIO WAVES (9 Hrs)

Modes of propagation , Structure of atmosphere , Ground wave propagation, Tropospheric propagation , Duct propagation, Troposcatter propagation , Flat earth and Curved earth concept Sky wave propagation – Virtual height, critical frequency , Maximum usable frequency – Skip distance, Fading , Multi hop propagation

OUTCOMES: **Total: 45 Hrs**

At the end of the course, the students should be able to:

- Explain the various types of antennas and wave propagation
- Write about the radiation from a current element
- Analyse the antenna arrays, aperture antenna and special antenna

TEXT BOOK:

1. John D Kraus," Antennas for all Applications", 4th Edition, McGraw Hill, 2010.

REFERENCES:

1. Edward C. Jordan and Keith G. Balmain" Electromagnetic Waves and Radiating Systems" Prentice Hall of India, 2nd Edition 2011.
2. R.E. Collin," Antennas and Radio wave Propagation", McGraw Hill 1985.
3. Constantine.A. Balanis "Antenna Theory Analysis and Design", Wiley Student Edition, 4th Edition 2016.
4. Rajeswari Chatterjee, "Antenna Theory and Practice" Revised Second Edition New Age International Publishers, 2006.
5. S. Drabowitch, "Modern Antennas" Second Edition, Springer Publications, 2007
6. Robert S. Elliott "Antenna Theory and Design" Wiley Student Edition, 2006.
7. H. Sizun "Radio Wave Propagation for Telecommunication Applications", First Indian Reprint, Springer Publications, 2007.

Subject Name: Antennas & Propagation

Topic Name: Fundamentals of Radiation

(Unit – 1)

Syllabus / Fundamentals of Radiation

1. Antenna Parameters
2. Radiation from oscillating current elements
3. Half wave dipole and folded dipole

Aim and Objective:

- To give insight of basic Knowledge of Antenna parameters
- To give thorough understanding of the radiation characteristics of Oscillating current elements
- To impart knowledge about half wave dipole and monopole

Pre Test – MCQ:

1. A permanent magnet

(a) attracts some substances and repels others

(b) attracts all paramagnetic substan¬ces and repels others

(c) attracts only ferromagnetic sub¬stances

(d) attracts ferromagnetic substances and repels all others

Ans: a

2. Magnetic moment is a

(a) pole strength

(6) universal constant

(c) scalar quantity

(d) vector quantity

Ans: d

3. Temporary magnets are used in

(a) loud-speakers

(b) generators

(c) motors

(d) all of the above

Ans: d

4. The unit of magnetic flux is

(a) henry

(b) weber

(c) ampereturn/weber

(d) ampere/metre

Ans: b

5. The law that the induced e.m.f. and current always oppose the cause producing them is due to

(a) Faraday

(b) Lenz

(c) Newton

(d) Coulomb

Ans: b

6.An antenna is a transitional structure between –

(a) Free space and guiding element
(b) Free space and free space
(c) Guiding element and another guiding element
(d) All of the above

Ans : a

7.The guiding device used for the antenna system –

(a) Transmission line
(b) Co-axial line
(c) Waveguide
(d) All of the above

Ans : d

8.If the radiation from an antenna is represented in terms of field strength, it is called –

(a) Field pattern
(b) Power pattern
(c) Radiation pattern
(d) None of the above

Ans: a

9.A major lobe is defined as the radiation lobe containing –

(a) Direction of minimum radiation
(b) Direction of maximum radiation
(c) Direction of moderate radiation
(d) None of the above

Ans : b

10.The First-null beam width (FNBW) is defined as the angular measurement between the directions –

(a) radiating the maximum power
(b) radiating half of the maximum power
(c) radiating no power
(d) None of the above

Ans : c

11.The half-power beam width (HPBW) is defined as the angular measurement between the directions –

(a) radiating the maximum power
(b) radiating half of the maximum power
(c) radiating no power
(d) None of the above

Ans : b

12.The x-z plane (elevation plane; ϕ = 0) is the principal –

(a) $\boldsymbol{E}$-plane
(b) H-plane
(c) Either E-plane or H-plane
(d) None of the above

Ans: a

13.An Omni directional antenna is a special antenna of type –

(a) Directional
(b) Non-directional
(c) Isotropic
(d) None of the above

Ans : a

14.The ratio of the main beam area to the total beam area is called –

(a) beam efficiency
(b) beam deficiency
(c) stray factor
(d) None of the above

Ans: a

15.Radiation density is defined as power per –

(a) Unit area
(b) Unit angle
(c) Unit field
(d) None of the above

Ans: a

Pre-requisite:

- Basic knowledge of Electromagnetic Fields and Wave guides.

UNIT I: FUNDAMENTALS OF RADIATION:

Definition of antenna parameters – Gain, Directivity, Effective aperture, Radiation Resistance, Band width, Beam width, Input Impedance. Matching – Baluns, Polarization mismatch, Antenna noise temperature, Radiation from oscillating dipole, Half wave dipole. Folded dipole, Yagi array.

INTRODUCTION - ANTENNA:

- An antenna is a transitional structure (metallic device: as a rod or wire) between free-space and a guiding device , for radiating or receiving radio waves as shown in Fig. 1-1.

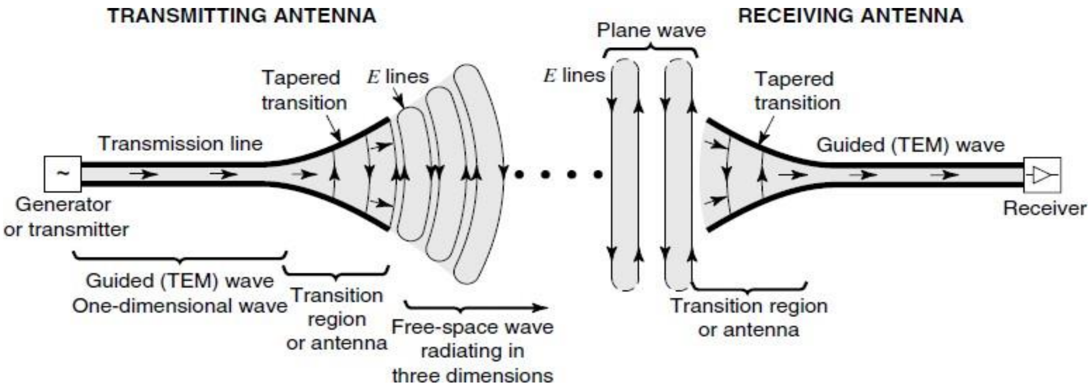


Fig. 1-1 : Wireless communication link with Transmitting and Receiving antenna

- The guiding device may take the form of a transmission line (coaxial line) or a waveguide (hollow pipe), and it is used to transport electromagnetic energy (from the transmitting source to the antenna, or from the antenna to the receiver).

- Thevenin equivalent of the antenna system in the transmitting mode is shown in Fig. 1-2. The source is represented by an ideal generator (with voltage V_g and impedance Z_g, [$Z_g = R_g + j\ X_g$]), the transmission line is represented by a line with characteristic impedance Z_c , and the antenna is represented by a load with impedance Z_A , [$Z_A = (R_{loss} + R_{rad}) + j\ X_A$] connected to the transmission line.

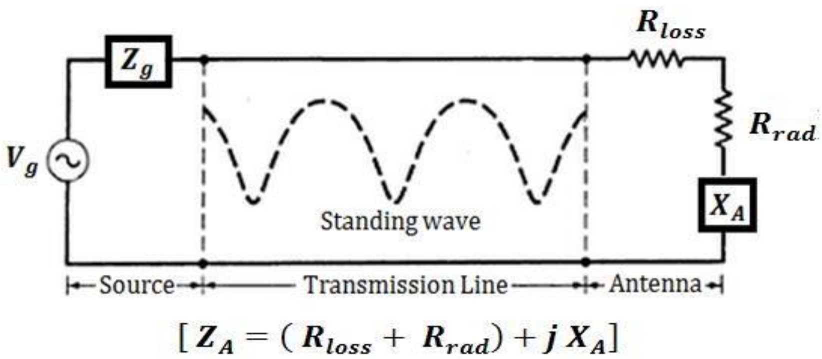


Fig. 1-2 Thevenin equivalent of antenna system in transmitting mode

- The load resistance R_{loss} represents the conduction and dielectric losses associated with the antenna while the radiation resistance R_{rad} represents radiation by the antenna. The reactance X_A represents the imaginary part of the impedance associated with radiation by the antenna.

DEFINITION OF ANTENNA PARAMETERS - RADIATION PATTERN

- An antenna radiation pattern is defined as "a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates".
- Radiation pattern is very important characteristic of an antenna. Radiation properties include radiation density, radiation intensity, field strength, gain, directivity, effective aperture, polarization, etc.,
- If the radiation from an antenna is represented in terms of field strength (electric or magnetic), then the radiation pattern is called ***field pattern***.
- Similarly, if the radiation from an antenna is represented in terms of power per unit solid angle, then the radiation pattern is called ***power pattern***.
- To completely specify the radiation pattern with respect to field intensity and polarization requires three patterns.
  - The θ component of the electric field as a function of the angles θ and ϕ or $E_\theta(\theta,\phi)$.
  - The ϕ component of the electric field as a function of the angles θ and ϕ or $E_\phi(\theta,\phi)$.
  - The phases of these fields as a function of the angles θ and ϕ or $\delta_\theta(\theta,\phi)$ and $\delta_\varphi(\theta,\phi)$.
- Fig. 1-2. shows three-dimensional field pattern (in spherical coordinates) of a directional antenna with maximum radiation in z- direction at $\theta = 0°$.

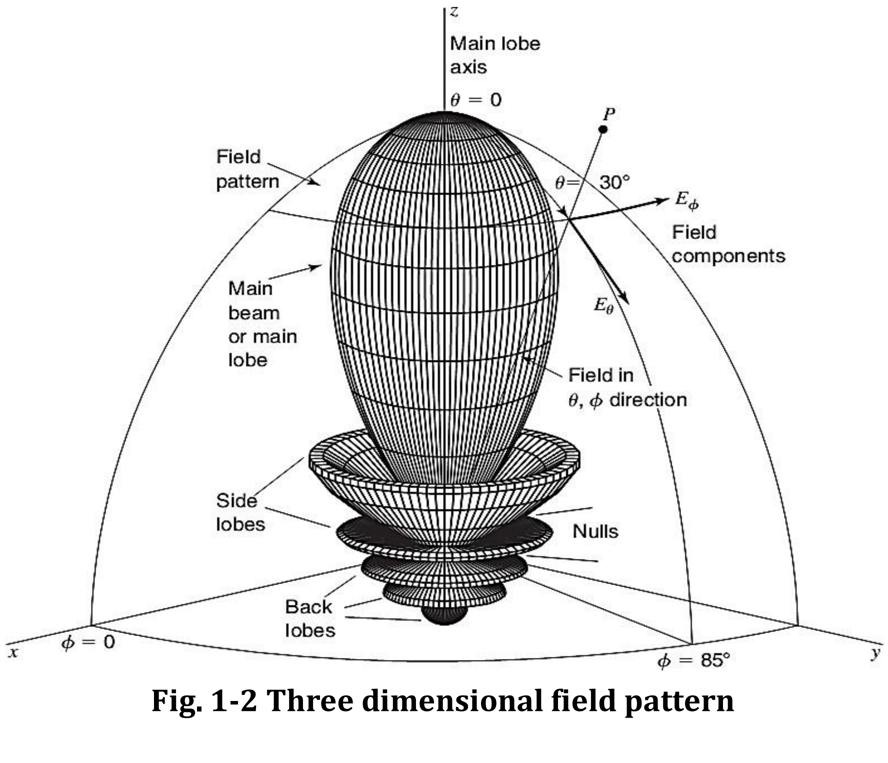


Fig. 1-2 Three dimensional field pattern

⇨ Radiation pattern lobes:

- Various parts of a radiation pattern are referred to as ***lobes***, which may be subclassified into major or main, minor, side and back lobes. A major lobe is defined as the radiation lobe containing the direction of maximum radiation.
- A minor lobe is any lobe except a major lobe. A side lobe is a radiation lobe in any direction other than the lobe. A back lobe is a radiation lobe whose axis makes an angle of approximately 180° with respect to the beam of an antenna.

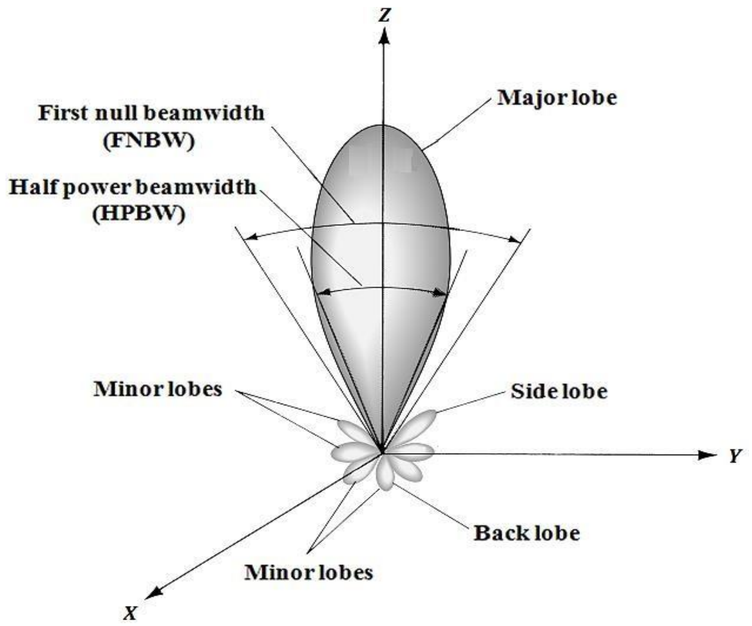


Fig. 1-3 Radiation lobes and beamwidths of an antenna

- Fig. 1-3 demonstrates a symmetrical three dimensional power pattern with a number of radiation lobes. Fig. 1-4 illustrates a linear plot of power pattern and its associated lobes and beamwidths.
- The ***half-power beamwidth*** (HPBW) is defined as the angular measurement between the directions in which the antenna is radiating half of the maximum value.
- The ***First-null beamwidth*** or ***beamwidth between first two nulls*** (FNBW) is defined as the angular measurement between the directions radiating no power.

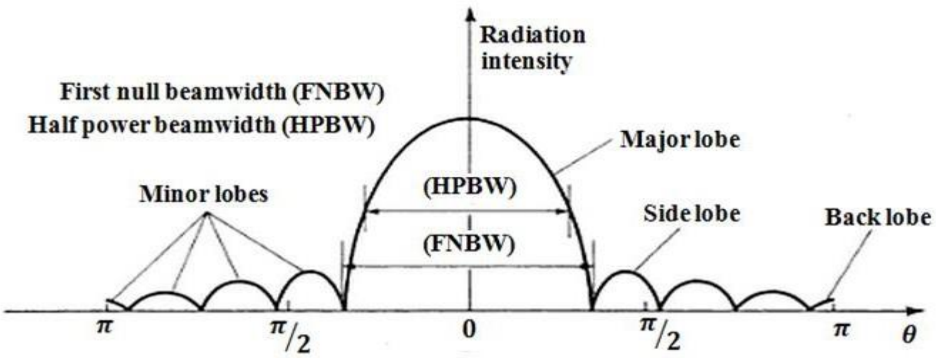


Fig. 1-4 Linear plot of power pattern and its associated lobes and beamwidths.

- Often the field and power patterns are normalized with respect to their maximum value, yielding normalized field and power patterns. Fig.1-5 shows the normalized field pattern and normalized power pattern.
- Dividing a field component by its maximum value, we obtain a *normalized* or *relative field pattern* which is a dimensionless number with maximum value of unity. The half power level occurs at those angles θ and ϕ for which $\boldsymbol{E_\theta(\theta,\phi)_n = 0.707}$ (or) $\boldsymbol{E_\phi(\theta,\phi)_n = 0.707}$.

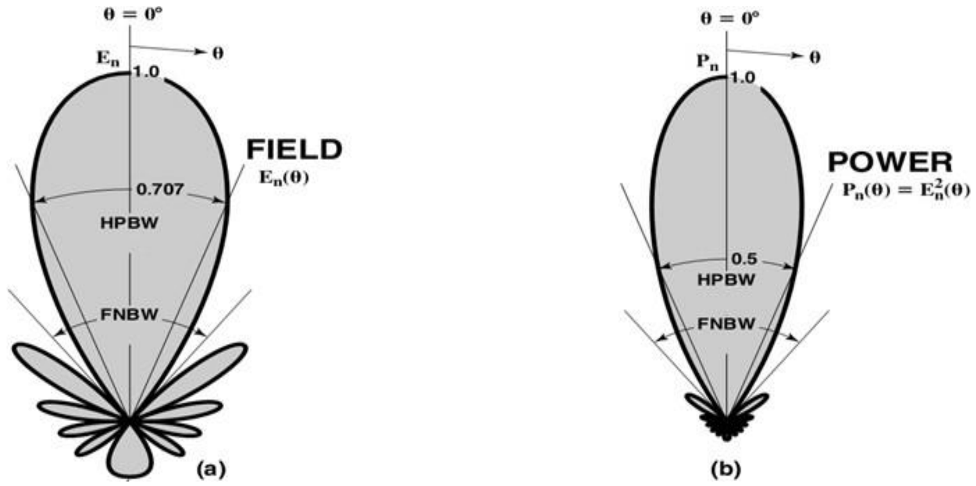


Fig.1-5 (a) Normalized field pattern (b) Normalized power pattern

- Thus the normalized field pattern for the electric field is given by;

$$\textbf{Normalized field pattern} = E_\theta(\theta,\phi)_n = \frac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{max}} \quad (or)$$

$$= E_\phi(\theta,\phi)_n = \frac{E_\phi(\theta,\phi)}{E_\phi(\theta,\phi)_{max}}$$

- Patterns may also be expressed in terms of the power per unit area. Normalizing this power with respect to its maximum value yields *normalized power pattern* as a function of angle which is a dimensionless number with a maximum value of unity. The half power level occurs at those angles θ and ϕ for which $P_n(\theta,\phi)_n = 0.5$.
- Thus the normalized power pattern is given by;

$$\textbf{Normalized power pattern} = P_n(\theta,\phi)_n = \frac{W(\theta,\phi)}{W(\theta,\phi)_{max}}$$

Where ; $W(\theta,\phi)$ = Poynting vector = $[E^2_\theta(\theta,\phi) + E^2_\phi(\theta,\phi)]/\eta_0$, (W/m^2)

$W(\theta,\phi)_{max}$ = maximum value of $W(\theta,\phi)$

η_0 = Intrinsic impedance of free space = 377 Ω

⇨ **Principle patterns:**

- For a linearly polarized antenna, performance is often described in terms of its principal $\boldsymbol{E}$-and $\boldsymbol{H}$-plane patterns. The $\boldsymbol{E}$-plane is defined as "the plane containing the electric-field vector and the direction of maximum radiation," and the $\boldsymbol{H}$-plane as "the plane containing the magnetic-field vector and the direction of maximum radiation."
- The x-z plane (elevation plane; $\phi = 0$) is the principal $\boldsymbol{E}$-plane and the x-y plane (azimuthal plane; $= \pi/2$) is the principal $\boldsymbol{H}$-plane.

ISOTROPIC, DIRECTIONAL AND OMNIDIRECTIONAL PATTERNS

- An ***isotropic radiator*** is defined as "a hypothetical lossless antenna having equal radiation in all directions." Although it is ideal and not physically realizable, it is often taken as a reference for expressing the directive properties of actual antennas.
- A ***directional antenna*** is one having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others.
- An ***omnidirectional antenna*** is defined as one "having an essentially non-directional pattern in a given plane and a directional pattern in any orthogonal plane. An omnidirectional pattern is then a special type of a directional pattern.

BEAM SOLID ANGLE (OR) BEAM AREA

- In polar-two dimensional coordinates an incremental area dA on the surface of a sphere is the product of the length $rd\theta$ in the θ direction and $r \sin\theta\, d\phi$ in the ϕ direction, as shown in Fig. 1-6.

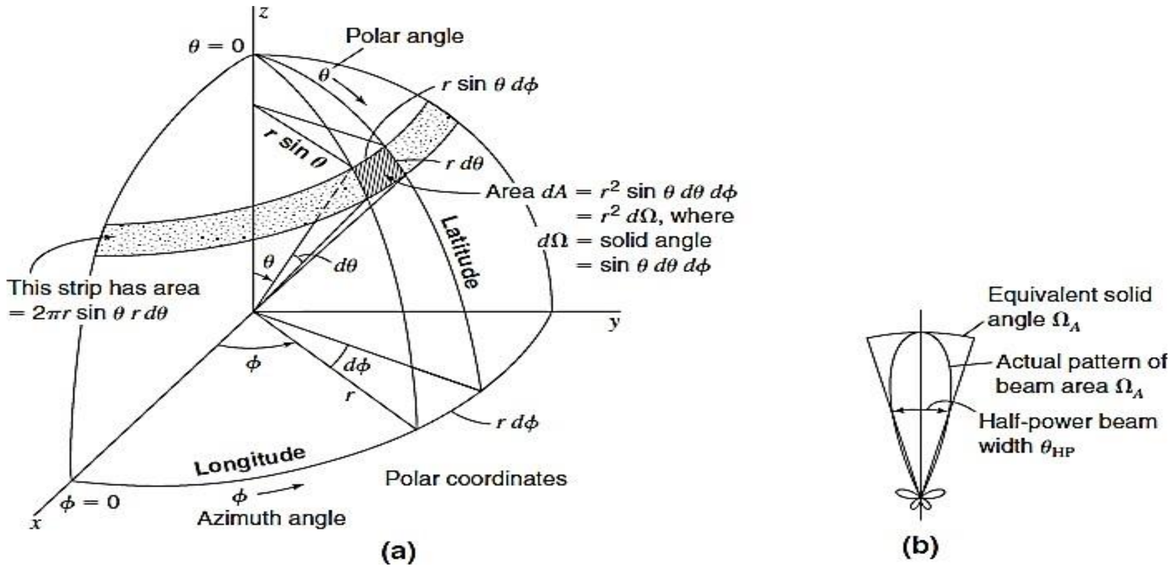


Fig.1-6 (a) Polar coordinates showing incremental solid angle $dA = r^2\, d\Omega$ on the surface of a sphere of radius r (b) Antenna power pattern and its equivalent solid angle.

- The incremental area of a sphere is given by;

$$dA = (\, rd\theta\,).(r \sin\theta\, d\phi) = r^2 \sin\theta\, d\theta d\phi \qquad \text{-------- (1.1)}$$

where $d\Omega$ = solid angle subtended by the area dA, and $\quad d\Omega = dA/r^2 \sin\theta\, d\theta d\phi$

(Generally solid angle is nothing but angle subtended by an elementary area on a sphere.)

- The beam solid angle of an antenna is given by the integral of the normalized power pattern over a sphere (4π).

$$\Omega_A = \iint_{\Omega=4\pi} P_n(\theta, \phi)\, d\Omega \qquad \text{-------- (1.2)}$$

where 4π = solid angle subtended by a sphere (**sr**, Sterdian)

$$\Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) \sin\theta\, d\theta d\phi \qquad \text{-------- (1.3)}$$

- The beam area of an antenna can often be described approximately in terms of the angles subtended by the half-power points of the main lobe in the two principal planes. Thus,

$$\Omega_A = \theta_{HP}\,\phi_{HP} \qquad \text{-------- (1.4)}$$

where θ_{HP} and ϕ_{HP} are the half-power beamwidths (HPBW) in the two principal planes, minor lobes being neglected.

- The (total) beam area consists of the main beam area (Ω_M) plus the minor-lobe area (Ω_m).

$$\Omega_A = \Omega_M + \Omega_m \qquad \text{-------- (1.5)}$$

- The ratio of the main beam area to the (total) beam area is called the (main) beam efficiency.

$$\varepsilon_M = \Omega_M/\Omega_A \qquad \text{-------- (1.6)}$$

- The ratio of the minor-lobe area to the (total) beam area is called the (main) stray factor.

$$\varepsilon_m = \Omega_m/\Omega_A \qquad \text{-------- (1.7)}$$

It follows that $\varepsilon_M + \varepsilon_m = 1$

RADIATION DENSITY

- Electromagnetic waves are used to transport information through a wireless medium or a guiding structure, from one point to the other. The quantity used to describe the power associated with an electromagnetic wave is the instantaneous Poynting vector defined as;

$$\boldsymbol{W} = \boldsymbol{E} \times \boldsymbol{H}$$

where ; $\boldsymbol{W} =$ instantaneous Poynting vector , $(\boldsymbol{W/m^2})$
$\boldsymbol{E} =$ instantaneous electric-field intensity, $(\boldsymbol{V/m})$
$\boldsymbol{H} =$ instantaneous magnetic-field intensity , $(\boldsymbol{A/m})$

- The total power crossing a closed surface can be obtained by integrating the normal component of the Poynting vector over the entire surface.

$$\boldsymbol{P} = \oiint_S \boldsymbol{W}.\,\boldsymbol{ds} \qquad \text{-------- (1.8)}$$

- For time-varying fields, the time average power density (average Poynting vector);

$$\boldsymbol{W_{rad}} = \frac{1}{2} Re[\boldsymbol{E} \times \boldsymbol{H}^*] \qquad \text{-------- (1.9)}$$

- The average power radiated by an antenna (radiated power) can be written as

$$P_{rad} = \oiint_S W_{rad}.\,ds = \oiint_S \frac{1}{2} Re[E \times H^*]\,.ds \qquad \text{-------- (1.10)}$$

RADIATION INTENSITY

- Radiation intensity in a given direction is defined as " the power radiated from an antenna per unit solid angle". The radiation intensity is a far-field parameter, and it can be obtained by simply multiplying the radiation density by the square of the distance. It is given by;

$$U = r^2\, W_{rad} \qquad \text{-------- (1.11)}$$

where ; $U =$ radiation intensity, (**W/unit solid angle**)
$W_{rad} =$ radiation density, $(\boldsymbol{W/m^2})$

- The total power radiated by the antenna is obtained by integrating the radiation intensity over the entire solid angle of 4π. Thus,

$$P_{rad} = \oiint_{\Omega} U(\theta,\phi)\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi \qquad \text{-------- (1.12)}$$

where $d\Omega$ =element of solid angle = $\sin\theta\, d\theta\, d\phi$

- For an isotropic source, U will be independent of the angles θ and ϕ. Thus (1.12) can be written as,

$$P_{rad} = \oiint_{\Omega} U_0\, d\Omega = U_0 \int_0^{2\pi}\int_0^{\pi} d\Omega = 4\pi U_0 \qquad \text{-------- (1.13)}$$

- The radiation intensity of an isotropic source as;

$$U_0 = P_{rad}/4\pi \qquad \text{-------- (1.14)}$$

<u>Note:</u> The normalized power pattern $P_n(\theta,\phi)_n$ can also be obtained by normalizing the radiation intensity $U(\theta,\phi)$.

Normalized power pattern: $$P_n(\theta,\phi)_n = \frac{U(\theta,\phi)}{U(\theta,\phi)_{max}}$$

$$U(\theta,\phi) = U(\theta,\phi)_{max}\, P_n(\theta,\phi)_n \qquad \text{-------- (1.15)}$$

DIRECTIVE GAIN & DIRECTIVITY

- Directive gain of an antenna is defined as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions".

$$D = \frac{U(\theta,\phi)}{U(\theta,\phi)_{av}} = \frac{U}{U_0} \qquad \text{-------- (1.16)}$$

- The average radiation intensity is equal to the total power radiated divided by 4π. In mathematical form ;

$$D = \frac{U}{(P_{rad}/4\pi)} = \frac{4\pi U}{P_{rad}} \qquad \text{-------- (1.17)}$$

- If the direction is not specified, it implies the direction of maximum radiation intensity (maximum directivity, D_0,) expressed as;

$$D_{max} = D_0 = \frac{4\pi U_{max}}{P_{rad}} \qquad \text{-------- (1.18)}$$

- ***Directivity*** (maximum directivity, D_0,) : It is defined as the ratio of the maximum radiation intensity to the average radiation intensity.

- The total power radiated by the antenna is defined in terms of normalized power pattern is gien by ; (Eqn. 1.12 in Eqn.1.15)

$$P_{rad} = U_{max} \oiint_{\Omega} P_n(\theta,\phi)_n\, d\Omega \qquad \text{-------- (1.19)}$$

Eqn. (1.19) in (1.18);

$$D_0 = \frac{4\pi U_{max}}{U_{max} \oiint_\Omega P_n(\theta,\phi)\, d\Omega_n} = \frac{4\pi}{\oiint_\Omega P_n(\theta,\phi)\, d\Omega_n}$$

$$\boxed{D_0 = \frac{4\pi}{\Omega_A} \quad (Directivity\ from\ Beam\ Area)}$$ -------- (1.20)

POWER GAIN

- Gain of an antenna is defined as "the ratio of the radiation intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to isotropically radiated power is equal to the power accepted (input) by the antenna divided by 4π."

$$G = 4\pi \frac{Radiation\ Intensity}{Total\ input\ (accepted)\ power} = 4\pi \frac{U(\theta,\phi)}{P_{in}}$$ -------- (1.21)

- In most cases we deal with relative gain, which is defined as " the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced direction."

- The power input must be the same for both antennas. The reference antenna is usually a dipole, horn or any other antenna whose gain can be calculated or it is known. However, the reference antenna is a *lossless isotropic source*. Thus,

$$G = 4\pi \frac{U(\theta,\phi)}{P_{in}(lossless\ isotropic\ source)}$$ -------- (1.22)

- The total power radiated, P_{rad}, by the antenna is related to the input power, P_{in}, by;

$$P_{rad} = \kappa P_{in}$$ -------- (1.23)

Sub Eqn. (1.23) in (1.21) ;

$$G = 4\pi \frac{U(\theta,\phi)}{\left(\frac{P_{rad}}{\kappa}\right)}$$

$$G(\theta,\phi) = \kappa D(\theta,\phi)$$ -------- (1.24)

where κ = antenna efficiency factor.

- The gain (G) of an antenna is an actual or realized quantity which is less than the directivity (D) due to ohmic losses in the antenna or its radome (if it is enclosed). The maximum value of the gain is related to maximum directivity;

$$G(\theta,\phi)_{max} = \kappa D(\theta,\phi)_{max}$$ -------- (1.25)

$$\boxed{G_0 = \kappa D_0}$$ -------- (1.26)

Note: When the direction is not stated, the power gain is usually taken in the direction of maximum radiation.

ANTENNA EFFICIENCY FACTOR

- Antenna Efficiency is defined as the ratio of power radiated by the antenna to the total input power supplied by the antenna. It is denoted by κ. It value lies between $0 \le \kappa \le 1$.

$$\frac{\text{Power radiated}}{\text{Total input power}}$$

$$\frac{P_{rad}}{P_{in}} \quad \frac{P_{rad}}{P_{rad} + P_{loss}}$$

$$\kappa = \frac{\frac{1}{2}|I|^2 R_{rad}}{\frac{1}{2}|I|^2 R_{rad} + \frac{1}{2}|I|^2 R_{loss}}$$

$$\frac{R_{rad}}{R_{rad} + R_{loss}} \qquad \text{-------- (1.27)}$$

INPUT IMPEDANCE

- Input impedance is defined as "the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point".

- Consider an antenna in the transmit mode, having an input impedance of $\boldsymbol{Z_A = R_A + j\, X_A}$, connected directly to a source having an equivalent Thevenin's voltage, $\boldsymbol{V_g}$, and an internal impedance $\boldsymbol{Z_g = R_g + j\, X_g}$, as shown in Fig.1-7.

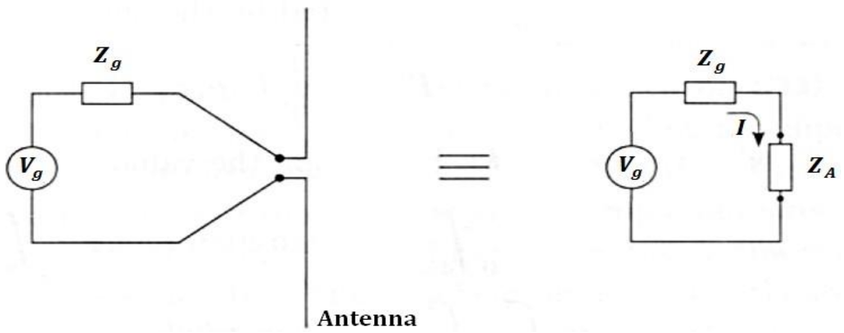


Fig. 1-7 Transmitting Antenna and its equivalent circuits.

where $\boldsymbol{Z_A}$ is the antenna impedance , $\boldsymbol{R_A}$ is the antenna resistance , X_A is the antenna reactance , $\boldsymbol{R_{rad}}$ is the radiation resistance of the antenna and $\boldsymbol{R_{loss}}$ is the ohmic loss resistance of the antenna.

- The maximum power transfer takes place when the antenna is conjugate-matched to the source, i.e.,

$$R_A = R_g \qquad X_A = -X_g \qquad \text{-------- (1.28)}$$

- Under the complex conjugate-match condition, the antenna input current is

$$I = \frac{V_g}{R_g + R_A} = \frac{V_g}{2R_A} \qquad \text{-------- (1.29)}$$

The real power supplied by the source is given by;

$$P_g = \frac{1}{2} Re\{V_g I^*\} = \frac{|V_g|^2}{4 R_A} \quad \text{-------- (1.30)}$$

- Half the power supplied by the source is lost in the source resistance, $\boldsymbol{R_g}$, and the other half gets dissipated in the antenna resistance, $\boldsymbol{R_A}$. Thus the effective power input to the antenna is ;

$$P_{in} = \frac{1}{2} |I|^2 R_A = \frac{|V_g|^2}{8R_A} \quad \text{-------- (1.31)}$$

- The antenna resistance, $\boldsymbol{R_A}$, is comprised of two components, namely the radiation resistance, $\boldsymbol{R_{rad}}$, and the loss resistance, $\boldsymbol{R_{loss}}$;

$$R_A = R_{rad} + R_{loss} \quad \text{-------- (1.32)}$$

- The total power inputed to the antenna is written as;

$$P_{in} = \frac{1}{2} |I|^2 R_A = \frac{1}{2} |I|^2 (R_{rad} + R_{loss}) \quad \text{-------- (1.33)}$$

- The total power radiated by an antenna is given by;

$$P_{rad} = \frac{1}{2} |I|^2 R_{rad} \quad \text{-------- (1.34)}$$

- The power dissipated in the antenna by ohmic losses is given by;

$$P_{loss} = \frac{1}{2} |I|^2 R_{loss} \quad \text{-------- (1.35)}$$

- For a matched antenna these are given by

$$P_{rad} = \frac{|V_g|^2}{8R_A} R_{rad} \quad \text{-------- (1.36)}$$

$$P_{loss} = \frac{|V_g|^2}{8R_A} R_{loss} \quad \text{-------- (1.37)}$$

⇨ **Concept of Self Impedance , Mutual Impedance And Transfer Impedance:**

- The impedance of antenna measured at the terminals where transmission line carrying RF power connected is called *antenna input impedance*. These terminals are nothing but feed points of the antenna, the impedance is also called *feed point impedance*.

- As the RF power carried by the transmission line from the transmitter, excites or drives the antenna, the antenna input impedance can be alternatively called *driving-point impedance*.

- When the antenna is losseless and isolated from and other objects, the impedance offered by antenna to the transmission line is represented by a 2-terminal network with impedance Z_A as in Fig. 1-8.

- With a losseless and isolated antenna, the antenna terminal impedance is same as the self impedance of the antenna. The self impedance have, in general, a real and an imaginary part. The real part is designated as the self resistance (radiation resistance) and the imaginary part is called the self reactance.
- In case there are near by objects, say several other antennas, the terminal impedance can still be replaced by a 2-terminal network.
- *Self impedance of the antenna is nothing but the impedance measured at input terminals of an antenna with all other antennas are isolated from it.*

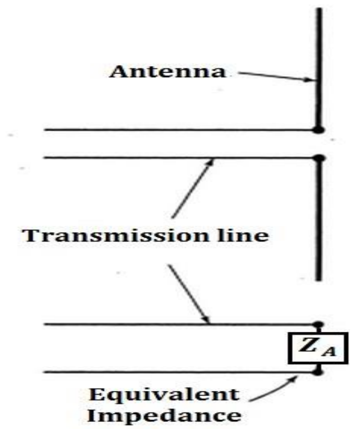


Fig. 1-8 Transmission line with antenna and with equivalent impedance.

- However, its value is determined not only by the self impedance of the antenna, but also by the mutual impedance between it and the other antennas and the currents flowing on them. The terminal is same for both transmission and reception.
- *The mutual impedance of the coupled circuit is defined as negative ratio of the voltage induced at the open terminals of one circuit to the current in other circuit.*

$$Z_{12} = -\frac{V_{12}}{I_2}; \qquad Z_{21} = -\frac{V_{21}}{I_1}$$

- The mutual impedance and the transfer impedance are altogether different concepts. While the transfer impedance is the ratio of voltage imposed in one circuit to the current in other circuit.
- In mutual impedance induced voltage is measured across open terminals while in transfer impedance current is measured in short circuited terminals.

<u>RADIATION RESISTANCE :</u>

- *Radiation resistance is the fictitious resistance such that when it is connected in series with an antenna will consume the same power as is actually radiated by the antenna.*
- The total power radiated by an antenna is given by;

$$P_{rad} = \frac{1}{2}|I|^2 R_{rad}$$

$$\therefore R_{rad} = \frac{P_{rad}}{\frac{1}{2}|I|^2}$$

Significance of radiation resistance:

The radiation resistance is the part of an antenna's feed point resistance that is caused by the radiation of electromagnetic waves from the antenna. The radiation resistance is determined by the geometry of the antenna. The energy lost by radiation resistance is converted to electromagnetic radiation.

BANDWIDTH:

- The Bandwidth of an antenna is defined as "the range of frequencies within which the performance of the antenna, with respect to some characteristics, conforms to a specified standard".
- The bandwidth can be considered to be the range of frequencies, on either side of a center frequency (usually the resonance frequency for a dipole), where the antenna characteristics (such as input impedance, pattern, beam width, polarization, side lobe level, gain, beam direction, beamwidth, radiation efficiency) are within an acceptable value of those at the center frequency.
- For narrow band antennas, the bandwidth is expressed as a percentage of the frequency difference (upper minus lower) over the center frequency of the bandwidth.
- In general, the antenna bandwidth mainly depends on impedance and pattern of antenna. At low frequency, generally impedance variation decides the bandwidth as pattern characteristics are frequency insensitive.
- Under such condition, bandwidth of the antenna is inversely proportional to Q factor of the antenna. The bandwidth of the antenna can be expressed as ;

$$\text{Bandwidth } (B.W) = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q} \qquad \text{-------- (1.38)}$$

$$\Delta f = f_2 - f_1 = \frac{f_0}{Q} \qquad \text{-------- (1.39)}$$

where f_0 is the center frequency or resonant frequency, while Q factor of antenna is given by,

$$Q = 2\pi = \frac{Total\ energy\ stored\ by\ antenna}{Energy\ radiated\ per\ cycle} \qquad \text{-------- (1.40)}$$

- Thus for lower Q antennas, the antenna bandwidth is very high and viceversa.

POLARIZATION:

- Polarization can be defined as the figure traced as a function of time by the tip of the instantaneous electric field vector at fixed location in free space observed along the direction of propagation.
- The polarization of an antenna is the polarization of the wave radiated by the antenna in the far-field. In the far-field region, the radiated field essentially has a spherical wavefront with ***E*** and ***H*** fields transverse to the radial direction, which is the direction of propagation.
- As the radius of curvature tends to infinity, the wavefront can be considered as a plane wave locally and the polarization of this plane wave is the polarization of the antenna.
- Generally the polarization of the antenna is direction-dependent, thus, i.e., polarization as a function of (θ, ϕ).
- Polarization of a plane wave describes the shape, orientation, and sense of rotation of the tip of the electric field vector as a function of time, looking in the direction of propagation.
- Consider a general situation in which the radiated electric field has both θ and ϕ components.

$$\boldsymbol{E} = \hat{\theta} E_\theta + \hat{\phi} E_\phi \qquad \text{-------- (1.41)}$$

where, $\boldsymbol{E_\theta}$ and $\boldsymbol{E_\phi}$ are functions of r, θ and ϕ can be complex. The instantaneous values of the electric field can be written as;

$$\bar{E}(r,\theta,\phi,t) = \hat{a}_\theta Re\,\{\boldsymbol{E_\theta}\, e^{j\omega t}\} + \hat{a}_\phi Re\,\{\boldsymbol{E_\phi}\, e^{j\omega t}\} \quad \text{-------- (1.42)}$$

- Let $\boldsymbol{E_\theta} = Ae^{j\alpha}$ and $\boldsymbol{E_\phi} = Be^{j\beta}$, where A and B are the magnitudes, α and β are the phase angles of $\boldsymbol{E_\theta}$ and $\boldsymbol{E_\phi}$, respectively. Substituting the above in Eqn. (1.42) and simplifying;

$$\bar{E}(r,\theta,\phi,t) = \hat{a}_\theta A\cos(\omega t + \alpha) + \hat{a}_\phi B\cos(\omega t + \beta) \quad \text{-------- (1.43)}$$

Depending on the values of A, B, α, and β in Eqn. (1.43), the tip of the $\boldsymbol{E}$ field vector can trace a straight line, a circle, or an ellipse. These three cases are termed as linear, circular, and elliptical polarizations and are explained in the following subsections.

⇨ Linear Polarization:

- A time-harmonic wave is linearly polarized at a given point in space if the electric field (or magnetic field) vector at that point is alweays oriented along the same straight line at every instant of time.

Condition: 1. Only one component,
2. Two orthogonal linear components are in time phase.

- Consider a field with $B = 0$ in Eqn. (1.43). Such an electric field has only a θ component. Then, the tip of the electric field traces a straight line along the $\hat{a}_\theta$-direction. On the other hand, if $A = 0$ and $B \neq 0$, the antenna is still linearly polarized but the orientation of the electric field is along $\hat{a}_\phi$.
- If A and B are not equal to zero and the θ and ϕ components of the electric field are in phase, i.e., if $\alpha = \beta$, the wave is still linearly polarized but the resultant vector is tilted with respect to $\hat{a}_\theta$ and the angle of the tilt depends on the A/B ratio. The plane of polarization makes an angle $\tan^{-1}(B/A)$ to the $\hat{a}_\theta$-direction.

⇨ Circular Polarization:

- A time-harmonic wave is circularly polarized at a given point in space if the electric field (or magnetic field) vector at that point traces a circle as a function of time.

Condition: 1. The field must have two orthogonal linear components,
2. The two components must have the same magnitude, and .
3. The two components must have a time-phase difference of odd multiples of 90°.

- If $A = B$ and $\beta = (\alpha - \pi/2)$ in Eqn. (1.43), the electric field vector is given by ;

$$\bar{E}(r,\theta,\phi,t) = \hat{a}_\theta A\cos(\omega t + \alpha) + \hat{a}_\phi B\sin(\omega t + \alpha) \quad \text{-------- (1.44)}$$

- As a function of time, the electric field vector traces a circle. The direction of rotation of the tip of the electric field vector is clockwise, looking in the direction of wave propagation, which is the positive r-direction. This can also be represented by a right-handed screw that moves along the direction of propagation if rotated clockwise. Such a wave is called *a right circularly polarized* (*RCP*) wave and the antenna is known as an *RCP* antenna.

- For $A = B$ and $\beta = (\alpha + \pi/2)$ in Eqn. (1.43), the electric field vector will still trace a circle but will rotate anticlockwise. Such a wave is called a *left circularly polarized* (LCP) wave and the antenna producing it would be an LCP antenna.

⇨ Elliptical Polarization:

- A time-harmonic wave is elliptically polarized at a given point in space if the electric field (or magnetic field) vector at that point traces an elliptical locus in space.

Condition:
1. The field must have two orthogonal linear components,
2. The two components can be of the same or different magnitude, and
3. i). If the two components are not of the same magnitude,the time-phase between two components must not be 0° or multiples of 180°(because it will be linear). ii). If the two components are of the same magnitude,the time-phase between two components must not be odd multiples of 90°(because it will be circular).

- In general, for $A \neq B \neq 0$ and $\alpha \neq \beta$, Eqn. (1.43), represents an elliptically polarized wave. At any point in space, the tip of the electric field of an elliptically polarized wave traces an ellipse as a function of time. The ratio of lengths of the major and minor axes of the ellipse is known as the axial ratio (AR).

$$AR = \frac{Length of the major axis}{Length of the minor axis} \qquad \text{-------- (1.45)}$$

- The orientation of the major axis with respect to the $\hat{a}$-axis is known as the tilt angle.
- Linear and circular polarizations are the two special cases of elliptical polarization with $AR = \infty$ and $AR = 1$,respectively.

Polarization Mismatch:

- In general, the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly stated as "polarization mismatch." The amount of power extracted by the antenna from the incoming signal will not be maximum because of the polarization loss.
- Assuming that the electric field of the incoming wave can be written as ;

$$\mathbf{E}_i = \hat{\boldsymbol{\rho}}_w E_i \qquad \text{-------- (1.46)}$$

where $\hat{\boldsymbol{\rho}}_w$ is the unit vector of the wave, and the polarization of the electric field of the receiving antenna can be expressed as

$$\mathbf{E}_a = \hat{\boldsymbol{\rho}}_a E_a \qquad \text{-------- (1.47)}$$

where $\hat{\boldsymbol{\rho}}_a$ is its unit vector (polarization vector), the polarization loss can be taken into account by introducing a *polarization loss factor* (PLF). It is defined, based on the polarization of the antenna in its transmitting mode, as

$$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = |\cos\psi_p|^2 \text{ (dimensionless)} \qquad \text{-------- (1.48)}$$

where ψ_p is the angle between the two unit vectors.

- The relative alignment of the polarization of the incoming wave and of the antenna is shown in Fig. 1-9. If the antenna is polarization matched, its PLF will be unity and the antenna will extract maximum power from the incoming wave.

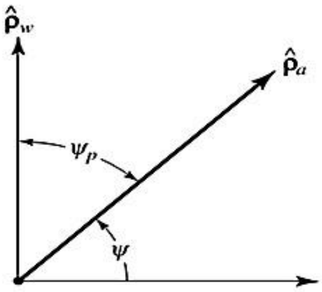


Fig. 1-9 Polarization unit vectors of incident wave ($\hat{\rho}_w$) and antenna ($\hat{\rho}_a$), and polarization loss factor (PLF).

- To illustrate the principle of polarization mismatch, the following example is considered.

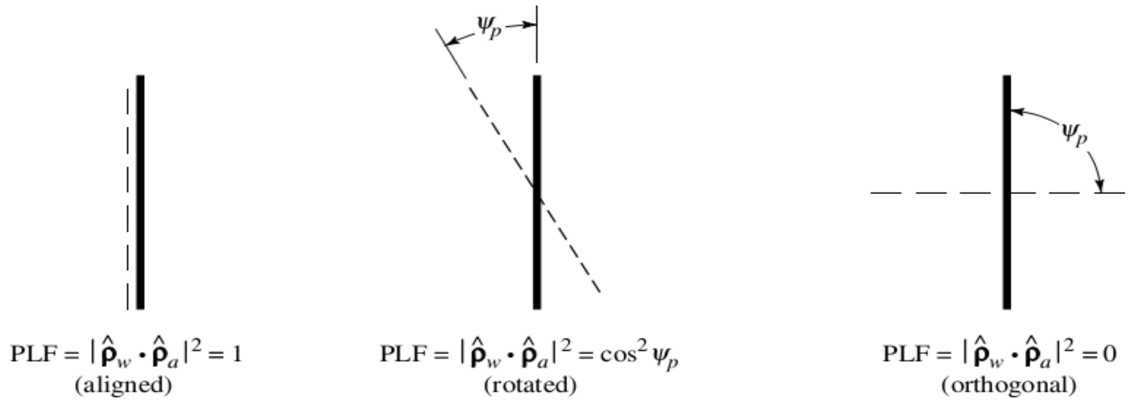


Fig. 1-10 PLF for transmitting and receiving linear wire antennas

- If the polarization of the incoming wave is orthogonal to the polarization of the antenna, then there will be no power extracted by the antenna from the incoming wave and the PLF will be zero or$-\infty$ dB. Fig. 1-10, illustrate the polarization loss factors (PLF) for linear wire antenna.

MATCHING – BALUNS

- Transmission lines are referred to as balanced or unbalanced. Parallel wire lines are inherently balanced in that if an incident wave (with balanced currents) is launched down the line, it will excite balanced currents on a symmetrical antenna.
- On the other hand, a coaxial transmission line is not balanced. A wave traveling down the coax may have a balanced current mode, that is, the currents on the inner conductor and the inside of the outer conductor are equal in magnitude and opposite in direction.
- However, when this wave reaches a symmetrical antenna, a current may flow back on the outside of the outer conductor, which unbalances the antenna and transmission line. This is illustrated in Fig. 1-11.
- Note that the currents on the two halves of the dipole are unbalanced. The current I_3 flowing on the outside of the coax will radiate. The currents I_1, and I_2 in the coax are shielded from the external world by the thickness of the outer conductor. They could actually be

unbalanced with no resulting radiation; it is the current on the outside surface of the outer conductor that must be suppressed.

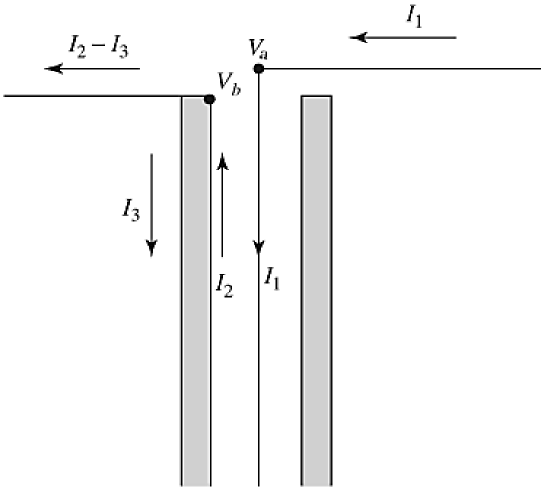


Fig. 1-11 Cross section of a coaxial transmission line feeding a dipole antenna at its center.

- To suppress this outside surface current, a balun (contraction for "balanced to unbalanced transformer") is used.
- The situation in Fig. 1-11 may be understood by examining the voltages that exist at the terminals of the antenna. These voltages are equal in magnitude but opposite in phase (i.e., $V_a = -V_b$). Both voltages act to cause a current to flow on the outside of the coaxial line.

- If the magnitude of the currents on the outside of the coax produced by both voltages are equal, the net current would be zero. However, since one antenna terminal is directly connected to the outer conductor, its voltage V_b, produces a much stronger current than the other voltage V_a.

- A balun is used to transform the balanced input impedance of the dipole to the unbalanced impedance of the coaxial line such that there is no net current on the outer conductor of the coax.

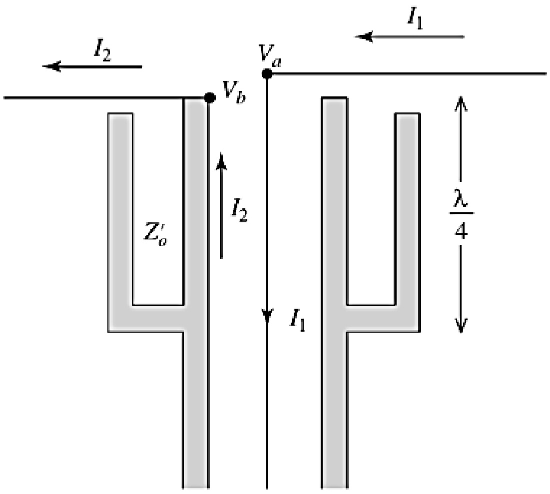


Fig. 1-12 Cross section of a sleeve balun feeding a dipole antenna at its center.

- One type of a balun is that shown in Fig. 1-12, referred to usually as a *bazooka* balun. Mechanically it requires that a $\lambda/4$ in length metal sleeve, and shorted at its one end, encapsulates the coaxial line.

Electrically the input impedance at the open end of this $\lambda/4$ shorted transmission line, which is equivalent to Z'_0, will be very large (ideally infinity). Thus the current I_3 will be choked, if not completely eliminated, and the system will be nearly balanced.

- Fig. 1-13 shows five common types of baluns. Type I balun (Fig. 1-13 (a)) has a $\lambda/4$ sleeve which provides an infinite impedance at the top. Type II balun (Fig. 1-13 (b)) has two Type I's in series providing high bandwidth and load balance at all frequencies. Type III balun (Fig. 1-13 (c)) is a more compact form.

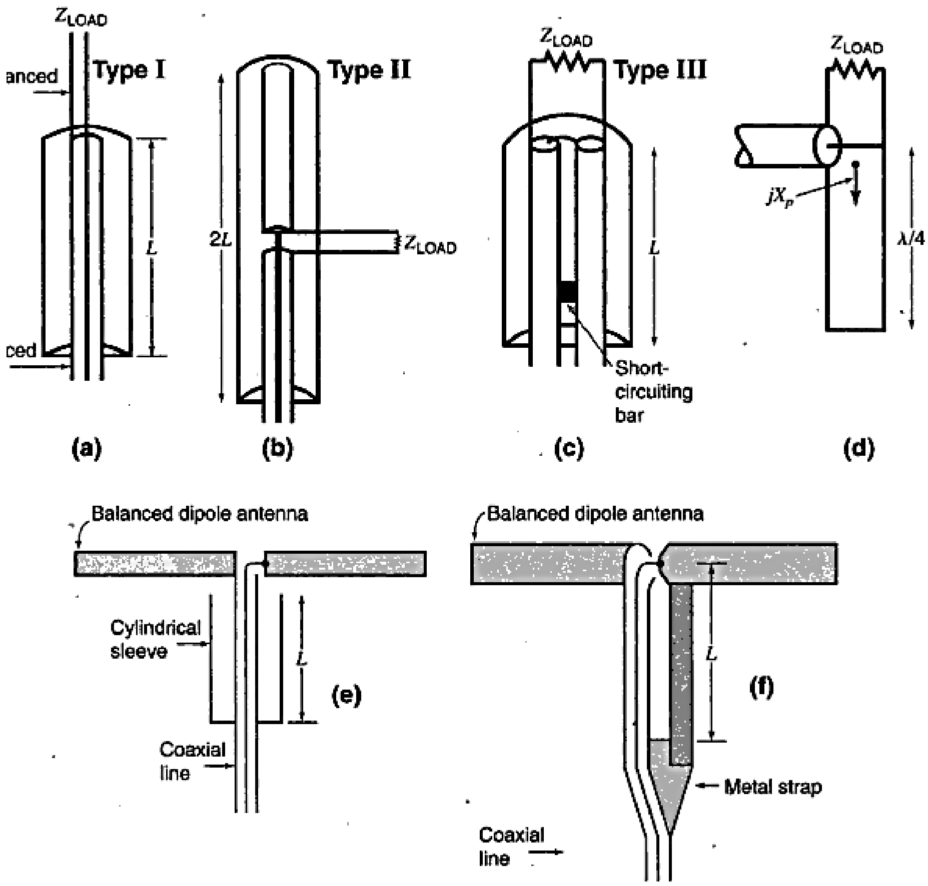


Fig. 1-13 (*a*) Type I balun or "bazooka," (*b*) Type II balun, (*c*) Type III balun, (*d*) Type III balun equivalent circuit, (*e*) Type I balun with dipole antenna and (*f*) dipole antenna with Type III balun minus sleeve.

- The inner conductors form a two conductor $\lambda/4$ line shorted at the base providing an infinite impedance at the top. It also features a sliding short circuit for frequency adjustment. Fig. 1-13 (d) shows the equivalent circuit of Type III balun.

- Type I balun with dipole is shown in Fig. 1-13 (e). Fig. 1-13 (f) has a dipole fed by a Type III balun minus shielding cavity. The length L of all baluns is about $\lambda/4$ at the center frequency. Baluns in Fig. 1-13 (e) & (f) provide a reactive impedance $Z = \pm Z_0 \tan \beta L$ in parallel with the dipole, where Z_0 is the characteristic impedance of the balun line of length L, and β the phase shift constant.

ANTENNA TEMPERATURE

- Every object with a physical temperature above absolute zero ($0K = -273°C$) radiates energy. The amount of energy radiated is usually represented by an equivalent temperature T_B, better known as brightness temperature, and it is defined as

$$T_B(\theta, \phi) = \xi(\theta, \phi) T_m \qquad \text{-------- (1.49)}$$

Where ; T_B = brightness temperature (equivalent temperature; K)

ξ = emissivity (dimensionless)

T_m = molecular (physical) temperature (K)

- Since the values of emissivity are $0 \le \xi \le 1$, the maximum value the brightness temperature can achieve is equal to the molecular temperature.
- Some of the better natural emitters of energy at microwave frequencies are (a) the ground with equivalent temperature of about 300 K and (b) the sky with equivalent temperature of about 5 K when looking toward zenith and about 100–150 K toward the horizon.

 The brightness temperature emitted by the different sources is intercepted by antennas, and it appears at their terminals as an antenna temperature. It is defined as;

$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi)\, G(\theta,\phi) \sin\theta \, d\theta \, d\varphi}{\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi) \sin\theta \, d\theta \, d\varphi} \quad \text{-------- (1.50)}$$

Where ; T_A = antenna temperature (effective noise temperature of the antenna radiation resistance ; K) , $G(\theta,\phi)$ = gain (power) pattern of the antenna.

 Assuming no losses or other contributions between the antenna and the receiver, the noise power transferred to the receiver is given by ;

$$P_r = kT_A\Delta f \quad \text{-------- (1.51)}$$

where ; P_r = antenna noise power (W)

k = Boltzmann's constant (1.38 × 10−23 J/K)

T_A = antenna temperature (K) Δf = receiver bandwidth (Hz)

 If the antenna and transmission line are maintained at certain physical temperatures, and the transmission line between the antenna and receiver is lossy, the antenna temperature T_A as seen by the receiver through (1.51) must be modified to include the other contributions and the line losses.

 If the antenna itself is maintained at a certain physical temperature T_p and a transmission line of length l, constant physical temperature T_0 throughout its length, and uniform attenuation of α (Np/m) is used to connect an antenna to a receiver, as shown in Fig. 1-14 , the effective antenna temperature at the receiver terminals is given by ;

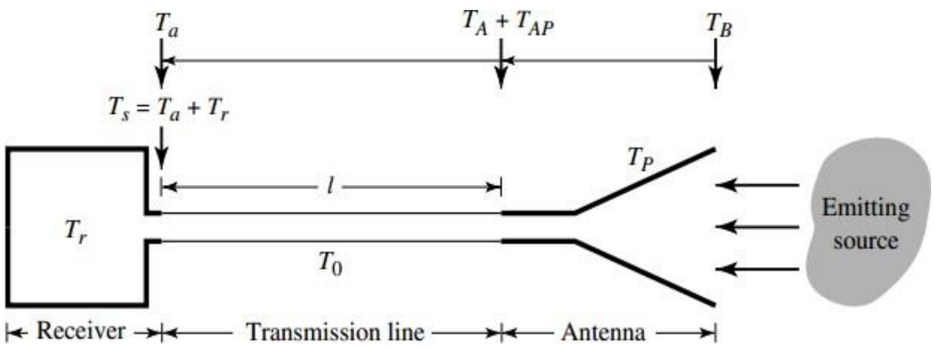


Fig. 1-14 Antenna, transmission line, and receiver arrangement for system noise power calculation.

$$T_a = T_A e^{-2\alpha l} + T_{AP} e^{-2\alpha l} + T_0(1 - e^{-2\alpha l}) \quad \text{-------- (1.52)}$$

$$T_{AP} = \left(\frac{1}{e_A} - 1\right) T_p$$

where ; T_a = antenna temperature at the receiver terminals

T_A = antenna noise temperature at the antenna terminals

T_{AP} = antenna temperature at the antenna terminals due to physical temperature

α = attenuation coefficient of transmission line

T_0 = physical temperature of the transmission line

T_p = antenna physical temperature

e_A = themal efficiency of antenna l= length of the transmission line

- The antenna noise power of Eqn. (1.52) must also be modified and written as;

$$P_r = kT_a\Delta f \qquad \text{-------- (1.53)}$$

- If the receiver itself has a certain noise temperature T_r(due to thermal noise in the receiver components), the system noise power at the receiver terminals is given by ;

$$P_s = k(T_a + T_r)\Delta f = kT_s\Delta f \qquad \text{-------- (1.54)}$$

where ; P_s = system noise power (at receiver terminals)

T_a = antenna noise temperature (at receiver terminals)

T_r = receiver noise temperature (at receiver terminals)

T_s = $T_a + T_r$ =effective system noise temperature (at receiver terminals)

<u>EFFECTIVE APERTURE</u>

- The effective aperture (also known as the effective area) of an antenna is the area over which the antenna collects energy from the incident wave and delivers it to the receiver load.
- If the power density in the wave incident from the (θ, ϕ) direction is $\boldsymbol{W}$ at the antenna and $P_r(\theta, \phi)$ is the power delivered to the load connected to the antenna, then the effective aperture, A_e,is defined as ;

$$A_e(\theta, \phi) = \frac{P_r(\theta, \phi)}{\boldsymbol{W}} \quad m^2 \qquad \text{-------- (1.55)}$$

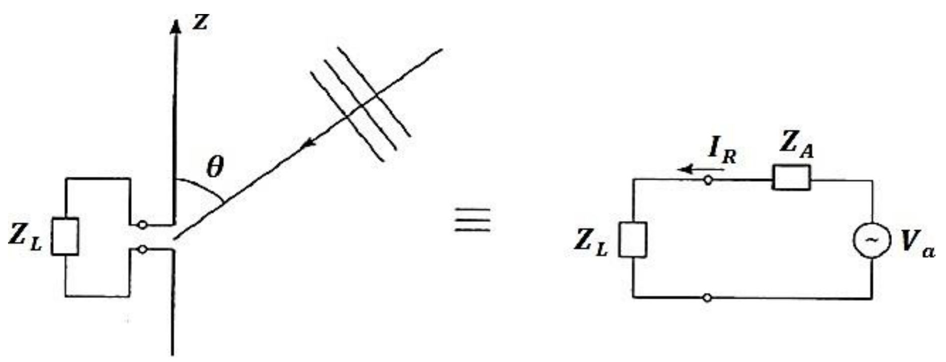


Fig. 1-15 shows the equivalent circuit of the receiving antenna and the load.

- Fig. 1-15 shows the equivalent circuit of the receiving antenna and the load, the power delivered to the load, Z_L, connected to the antenna terminals is ;

$$P_r = \frac{1}{2}|I_R|^2 R_L \qquad \text{-------- (1.56)}$$

where ; P_r = Power delivered to the load Z_L

R_L= Real part of the load impedance, ($Z_L = R_L + j\,X_L$)

- Let $Z_A = R_A + j\ X_A$ be the antenna impedance and V_a be Thevenin's equivalent source corresponding to the incident plane wave. The real part of the antenna impedance can be further divided into two parts, i.e., $R_A = R_{rad} + R_{loss}$.
- If the antenna is conjugate-matched to the load so that maximum power can be transferred to the load, we have $Z_L = Z_A{}^*$ or $R_L = R_A$ and $X_L = -X_A$.
- It is seen from the equivalent circuit that the power collected from the plane wave is dissipated in the three resistances, the receiver load, the radiation resistance, and the loss resistance.
- For a conjugate-match, the current through all three resistances is ;

$$I = \frac{V_a}{R_L + R_{rad} + R_{loss}} = \frac{V_a}{R_L + R_A} = \frac{V_a}{2R_L} \qquad \text{-------- (1.57)}$$

and the three powers are computed using the formulae ;

$$P_r = \frac{1}{2}|I|^2 R_L = \frac{1}{2}\frac{|V_a|^2}{(2R_L)^2} R_L = \frac{|V_a|^2}{8R_L} \qquad \text{-------- (1.58)}$$

$$P_{scat} = \frac{1}{2}|I|^2 R_{rad} = \frac{|V_a|^2}{8{R_L}^2}R_{rad} \qquad \text{-------- (1.59)}$$

$$P_{loss} = \frac{1}{2}|I|^2 R_{loss} = \frac{|V_a|^2}{8{R_L}^2}R_{loss} \qquad \text{-------- (1.60)}$$

where P_r is the power delivered to the receiver load, P_{loss} is the power dissipated in the antenna, and P_{scat} is the power scattered, since there is no physical resistance corresponding to the radiation resistance. The total power collected by the antenna is the sum of the three powers

$$P_c = P_r + P_{scat} + P_{loss} \qquad \text{-------- (1.61)}$$

- If the power density in the incident wave is $\boldsymbol{W}$, then the effective collecting aperture, A_c,of the antenna is the equivalent area from which the power is collected

$$A_c(\theta,\phi) = \frac{P_c(\theta,\phi)}{W} \quad m^2 \qquad \text{-------- (1.62)}$$

- This area is split into three parts $-A_e$: the effective aperture corresponding to the power delivered to the receiver load, A_{loss}: the loss aperture corresponding to the power loss in the antenna, and A_s: the scattering aperture corresponding to the power re-radiated or scattered by the antenna. These are given by

$$A_e(\theta,\phi) = \frac{P_r(\theta,\phi)}{W} = \frac{|V_a|^2}{8R_L W} \quad m^2 \qquad \text{-------- (1.63)}$$

$$A_s(\theta,\phi) = \frac{P_{scat}(\theta,\phi)}{W} = \frac{1}{2}|I|^2 R_{rad} = \frac{|V_a|^2}{8R_L^2 W} R_{rad} \quad \text{-------- (1.64)}$$

$$A_{loss}(\theta,\phi) = \frac{P_{loss}(\theta,\phi)}{W} = \frac{1}{2}|I|^2 R_{loss} = \frac{|V_a|^2}{8R_L^2 W} R_{loss} \quad \text{-------- (1.65)}$$

All these are functions of the incident direction (θ, ϕ).

⇨ ***Relationship between Directivity and effective aperture***

- Consider now an antenna with an effective aperture A_e, which radiates all of its power in a conical pattern of beam area Ω_A, as suggested in Fig. 1-16 .

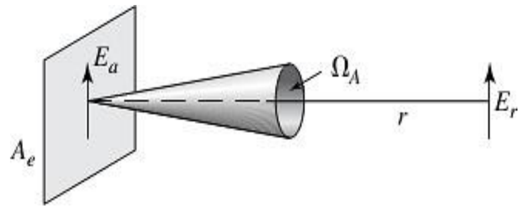


Fig. 1-16 Radiation over beam area Ω_A from aperture A_e

- Assuming a uniform field $\boldsymbol{E_a}$, over the aperture, the power radiated is ;

$$P = \frac{\boldsymbol{E_a}^2}{\eta} A_e \quad \text{-------- (1.66)}$$

- Assuming a uniform field $\boldsymbol{E_r}$ in the farfield at a distance r, the power radiated is also given by

$$P = \frac{\boldsymbol{E_r}^2}{\eta} r^2 \Omega_A \quad \text{-------- (1.67)}$$

where ; $\boldsymbol{E_r} = (\boldsymbol{E_a} A_e r)/\boldsymbol{\lambda}$

- Eqn. (1.66) and (1.67) yield aperture-beam area relationship ;

$$\boldsymbol{\lambda}^2 = A_e \Omega_A \quad \text{-------- (1.68)}$$

- We know that directive gain (D) and beam-area (Ω_A) relationship ;

$$D = \frac{4\pi}{\Omega_A} \quad \text{-------- (1.69)}$$

- Comparing the above equations ;

$$A_e = \frac{\boldsymbol{\lambda}^2}{4\pi} D$$

- Maximum effective aperture

$$\boxed{A_{em} = \frac{\boldsymbol{\lambda}^2}{4\pi} D_0} \quad \text{-------- (1.70)}$$

REVIEW OF ELECTROMAGNETIC THEORY:

- Electromagnetic fields are produced by time-varying charge distributions which can be supported by time-varying current distributions.
- Consider sinusoidally varying electromagnetic sources. (Sources having arbitrary variation with respect to time can be represented in terms of sinusoidally varying functions using

Fourier analysis). A sinusoidally varying current $i(t)$ can be expressed as a function of time, t, as:

$$i(t) = I_0 \cos(\omega t + \varphi) \quad \text{-------- (1.71)}$$

where I_0 is the amplitude (magnitude) of the current , ω is the angular frequency ($\omega = 2\pi f$) and φ is the phase angle.

- Eqn. (1.71) in phasor form:

$$i(t) = Re\ \{I_0 e^{j(\omega t+\varphi)}\} = Re\ \{I_0 e^{j\varphi} e^{j\omega t}\} \quad \text{-------- (1.72)}$$

where $I_0 e^{j\varphi}$ is a phasor that contains the amplitude and phase information of $i(t)$.

- Similarly, a sinusoidally varying instantaneous field vectors, $\bar{E}(x, y, z, t)$, can also be represented by phasors.

$$\bar{E}(x, y, z, t) = Re\ \{\ \boldsymbol{E}(x, y, z) e^{j\omega t}\} \quad \text{-------- (1.73)}$$

where $\boldsymbol{E}(x, y, z)$ is a phasor that contains the direction, magnitude and phase information of the electric field.

⇨ **Maxwells Equations:**

- Using phasor notation, Maxwell's Equations can be written for the fields and sources that are sinusoidally varying with time as,

$$\nabla \times \boldsymbol{E} = -j\omega \boldsymbol{B} = -j\omega\mu \boldsymbol{H} \quad \text{-------- (1.74)}$$

$$\nabla \times \boldsymbol{H} = j\omega \boldsymbol{D} + \boldsymbol{J} = j\omega\epsilon \boldsymbol{E} + \boldsymbol{J} \quad \text{-------- (1.75)}$$

$$\nabla . \boldsymbol{D} = \rho \quad \text{-------- (1.76)}$$

$$\nabla . \boldsymbol{B} = 0 \quad \text{-------- (1.77)}$$

- The symbols used in Eqns. (1.74) to (1.77) are explained below:

$\boldsymbol{E}$: Electric field intensity	$\boldsymbol{D}$: Electric flux density	$\boldsymbol{\rho}$: Charge density
$\boldsymbol{H}$: Magnetic field intensity	$\boldsymbol{B}$: Magnetic flux density	$\boldsymbol{J}$: Conduction current density

- The conduction current density, $\boldsymbol{J}$, is given by ;

$$\boldsymbol{J} = \sigma \boldsymbol{E} \quad \text{-------- (1.78)}$$

where σ is electric conductivity of the medium. For free space medium $\sigma = 0$.

- In an isotropic and homogeneous medium, the electric flux density, $\boldsymbol{D}$, and the electric field intensity, $\boldsymbol{E}$, are related by

$$\boldsymbol{D} = \epsilon \boldsymbol{E} \quad \text{-------- (1.79)}$$

where : $\epsilon = \epsilon_r \epsilon_0$ is electric permittivity of the medium, ϵ_0 is the permittivity of free space ($\epsilon_0 = 8.854 \times 10^{-12}$, $\boldsymbol{F/m}$) and ϵ_r is the relative permittivity of the medium.

- Similarly, magnetic flux density, $\boldsymbol{B}$, and magnetic field intensity, $\boldsymbol{H}$, are related by

$$\boldsymbol{B} = \mu \boldsymbol{H} \quad \text{-------- (1.80)}$$

where $\mu = \mu_r \mu_0$ is electric permeability of the medium, μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}$, $\boldsymbol{H/m}$) and μ_r is the relative permeability of the medium.

VECTOR POTENTIAL:

- Considering the current distribution in the antenna, the problem is to determine the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields due to this current distribution, which satisfy all the four Maxwell's equations along with the boundary conditions.

- In the vector potential approach we carry out the solution to this problem in two steps by defining intermediate potential functions (Fig. 1-17).
    i. In the first step, we determine the potential function due to the current distribution.
    ii. In the second step, the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are computed from the potential function.

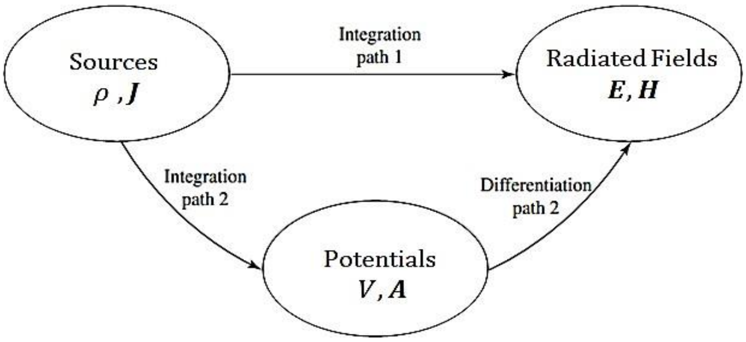


Fig. 1-17 Block diagram for computing fields radiated by electric and magnetic sources.

- Consider Maxwell's fourth equations, $\nabla . \boldsymbol{B} = 0$. Since the curl of a vector is divergence-free ($\nabla . \nabla \times \boldsymbol{A} = 0$), $\boldsymbol{B}$ can be expressed as a curl of an arbitrary vector field, $\boldsymbol{A}$.

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad \text{-------- (1.81)}$$

where $\boldsymbol{A}$ is the *magnetic vector potential function*, because it is related to the magnetic flux density, $\boldsymbol{B}$.

Eqn. (1.80) in Eqn. (1.81): $\mu \boldsymbol{H} = \nabla \times \boldsymbol{A}$

$$\boldsymbol{H} = \frac{\mathbf{1}}{\mu}(\nabla \times \boldsymbol{A}) \qquad \text{-------- (1.82)}$$

Eqn. (1.82) in Eqn. (1.74) (Maxwell's first equation $\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}$):

$$\nabla \times \boldsymbol{E} = -j\omega\mu \left[\frac{\mathbf{1}}{\mu}(\nabla \times \boldsymbol{A})\right] = -j\omega(\nabla \times \boldsymbol{A})$$

$$\nabla \times (\boldsymbol{E} + j\omega\boldsymbol{A}) = 0 \qquad \text{-------- (1.83)}$$

- Since the curl of a gradient is zero ($\nabla \times (-\nabla V) = 0$), the quantity $(\boldsymbol{E} + j\omega\boldsymbol{A})$ can be replaced by the gradient of a scalar function.

$$(\boldsymbol{E} + j\omega\boldsymbol{A}) = -\nabla V$$

$$\boldsymbol{E} = -(\nabla V + j\omega\boldsymbol{A}) \qquad \text{-------- (1.84)}$$

where V is the *electric scalar potential function.*

Eqns. (1.82) and (1.84) relate the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields to the potential functions $\boldsymbol{A}$ and V. (The fields expressed in Eqns. (1.82) and (1.84) satisfy two of Maxwell's equations (1.74) and (1.77)).

- To satisfy Maxwell's second equation ($\nabla \times \boldsymbol{H} = j\omega\epsilon\boldsymbol{E} + \boldsymbol{J}$), substitute Eqns. (1.82) and (1.84) in (1.75):

$$\nabla \times [\frac{1}{\mu}(\nabla \times \boldsymbol{A})] = j\omega\epsilon[-(\nabla V + j\omega\boldsymbol{A})] + \boldsymbol{J}$$

$$\nabla \times \nabla \times \boldsymbol{A} = -j\omega\mu\epsilon\nabla V + \omega^2\mu\epsilon\boldsymbol{A} + \mu\boldsymbol{J} \qquad \text{-------- (1.85)}$$

Using the vector identity $\nabla \times \nabla \times \boldsymbol{A} = \nabla(\nabla.\mathbf{A}) - \nabla^2\boldsymbol{A}$

$$\nabla(\nabla.\mathbf{A}) - \nabla^2\boldsymbol{A} = -j\omega\mu\epsilon\nabla V + \omega^2\mu\epsilon\boldsymbol{A} + \mu\boldsymbol{J}$$

$$\nabla^2\boldsymbol{A} + \omega^2\mu\epsilon\boldsymbol{A} = -\mu\boldsymbol{J} + \nabla(\nabla.\mathbf{A}) + j\omega\mu\epsilon\nabla V$$

$$\nabla^2\boldsymbol{A} + \omega^2\mu\epsilon\boldsymbol{A} = -\mu\boldsymbol{J} + \nabla(\nabla.\mathbf{A} + j\omega\mu\epsilon V) \qquad \text{-------- (1.86)}$$

To simplify Eqn. (1.86), the potential functions $\boldsymbol{A}$ and V are related by the equation;

$$\nabla.\mathbf{A} = -j\omega\mu\epsilon V \qquad \text{-------- (1.87)}$$

This relation is called *Lorentz condition.* With this the magnetic vector potential function, $\boldsymbol{A}$, satisfies the vector wave equation ;

$$\nabla^2\boldsymbol{A} + \omega^2\mu\epsilon\boldsymbol{A} = -\mu\boldsymbol{J} \qquad [\because k = \omega\sqrt{\mu\epsilon}]$$

$$\boxed{\nabla^2\boldsymbol{A} + k^2\boldsymbol{A} = -\mu\boldsymbol{J}} \qquad \text{-------- (1.88)}$$

where k is the propagation constant ($\boldsymbol{rad/m}$) in the medium.

- Now, to satisfy Maxwell's third equation ($\nabla.\boldsymbol{D} = \rho$), substitute Eqn. (1.84) in (1.76):

$$\nabla.\boldsymbol{D} = \rho \qquad [\because \boldsymbol{D} = \epsilon\boldsymbol{E}]$$

$$\nabla.\boldsymbol{E} = \frac{\rho}{\epsilon}$$

$$\nabla.(-(\nabla V + j\omega\boldsymbol{A})) = \frac{\rho}{\epsilon}$$

$$\nabla^2 V + j\omega(\nabla.\boldsymbol{A}) = -\frac{\rho}{\epsilon} \qquad \text{-------- (1.89)}$$

Eliminating $\boldsymbol{A}$ from the equation using the *Lorentz condition* [Eqn.(1.87)];

$$\boxed{\nabla^2 V + k^2 V = -\frac{\rho}{\epsilon}} \qquad \text{-------- (1.90)}$$

Thus, both $\boldsymbol{A}$ and V must satisfy the wave equation, the source function being the current density for the magnetic vector potential, $\boldsymbol{A}$, and the charge density for the electric scalar potential, V.

- The solutions of the Eqn's. (1.88) & (1.90) for time varying fields are given by ;

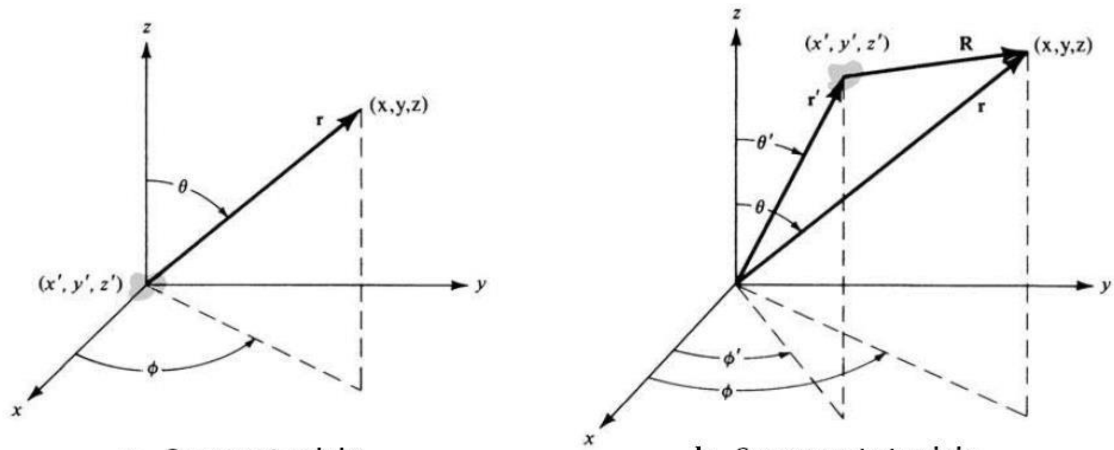
$$\boldsymbol{A} = \frac{\mu}{4\pi}\iiint_{V'} \frac{[\boldsymbol{J}]}{R}dv' \qquad \text{-------- (1.91)}$$

$$V = \frac{1}{4\pi\epsilon}\iiint_{V'} \frac{[\rho]}{R}dv' \qquad \text{-------- (1.92)}$$

- The term $[\rho]$ or $([\boldsymbol{J}])$ means that the time t in $\rho(x, y, z, t)$ or $[\boldsymbol{J}(x, y, z, t)]$ is replaced by the retarded time t', given by :

$$t' = t - \frac{R}{c} \qquad \text{-------- (1.93)}$$

where : c is the velocity of light , $R = |r - r'|$ is the distance between the source point r' and the observation point r as shown in Fig. 1-18.



a. Source at origin   b. Source not at origin

Fig.1-18 Coordinate system for computing fields radiated by sources

<u>Note:</u> The primed coordinates r' i.e., (x', y', z'), represents the source point and unprimed coordinates r i.e., (x, y, z) , represents the field point.

RETARDED POTENTIAL:

- It is clear that, the potential at time, t, is due to the source that existed at an earlier time R/c. Or the effect of any change in the source has travelled with a velocity c to the observation point at a distance R from the source. Therefore, $\boldsymbol{A}$ is also known as the ***retarded magnetic vector potential.*** It is given by;

$$\boldsymbol{A} = \frac{\mu}{4\pi} \iiint_{V'} \frac{\boldsymbol{J} e^{j\omega[t-(R/c)]}}{r} dv' \qquad Wb\,m^{-1} \qquad \text{-------- (1.94)}$$

- Similarly, the ***retarded electric scalar potential,*** V is given by;

$$V = \frac{1}{4\pi\epsilon} \iiint_{V'} \frac{\rho e^{j\omega[t-(R/c)]}}{R} dv' \qquad V \qquad \text{-------- (1.95)}$$

⇨ **<u>Phaor Form:</u>**

- Very often, sources of an electromagnetic field are time-harmonic. For harmonic time dependence the *phasor retarded potentials* are;

$$\boldsymbol{A}(x, y, z) = \frac{\mu}{4\pi} \iiint_{V'} \boldsymbol{J}(x', y', z') \frac{e^{-jkR}}{R} dv'$$ ***phasor retarded vector potential)***

$$V(x, y, z) = \frac{1}{4\pi\epsilon} \iiint_{V'} \rho(x', y', z') \frac{e^{-jkR}}{R} dv'$$ ***phasor retarded scalar potential)***

- If the current density is confined to a surface with current density, $\boldsymbol{J_s}$,the volume integral reduces to surface integral;

$$\boldsymbol{A}(x,y,z) = \frac{\mu}{4\pi} \iint_{S'} \boldsymbol{J_s}(x',y',z') \frac{e^{-jkR}}{R} ds' \qquad \text{-------- (1.96)}$$

- For a line current, $\boldsymbol{I}$, the integral reduces to surface integral;

$$\boldsymbol{A}(r) = \frac{\mu}{4\pi} \int_{C'} \boldsymbol{I}(x',y',z') \frac{e^{-jkR}}{R} dl' \qquad \text{-------- (1.97)}$$

<u>RADIATION FROM OSCILLATING DIPOLE (HERTZIAN DIPOLE)</u>

- A Hertzian dipole is "an elementary source consisting of a time-harmonic electric current element of a specified direction and infinitesimal length ($l \ll \lambda$)." It is also called infinitesimal dipole. It is also called as *alternating current element* or *oscillating currrnt element*.
- Although such a current element does not exist in real life, it serves as a building block from which the field of a practical antenna can be calculated by integration.
- Consider an infinitesimal time-harmonic current element, $\boldsymbol{I} = \hat{\boldsymbol{a}} I_0 dl$, kept at the origin with the current flow directed along the z-direction indicated by the unit vector $\hat{\boldsymbol{a}}$, as shown in Fig. 1-19. I_0 is the current and dl is the elemental length of the current element.

⇨ **<u>Radiated fields:</u>**

- To find the fields radiated by the current element, consider the following procedure.
  i. Determine the current distribution on the antenna structure and then compute the vector potential, $\boldsymbol{A}$. In a source-free region $\boldsymbol{A}$, is related to $\boldsymbol{H}$ field viz., Eqn. (1.82):

$$\boldsymbol{H} = \frac{\boldsymbol{1}}{\mu}(\nabla \times \boldsymbol{A}) \qquad \text{-------- (1.98)}$$

  ii. Then, $\boldsymbol{H}$ is related to $\boldsymbol{E}$ field viz., Eqn. (1.75) with $\boldsymbol{J} = 0$ in a source-free region.

$$\boldsymbol{E} = \frac{\boldsymbol{1}}{j\omega\epsilon}(\nabla \times \boldsymbol{H}) \qquad \text{-------- (1.99)}$$

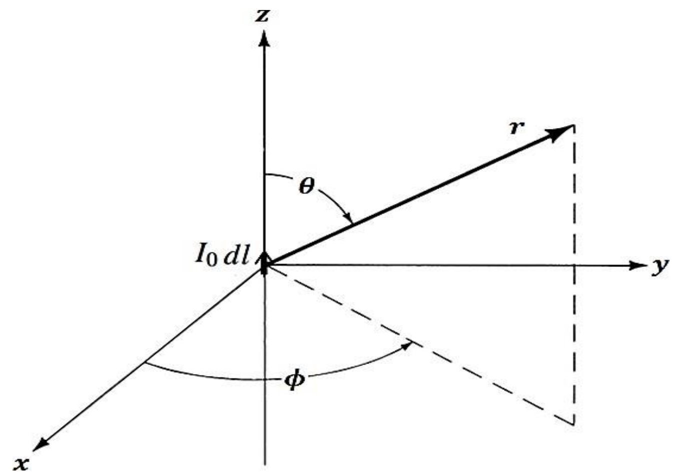


Fig. 1-19 Infinitesimal dipole

- Consider the relationship between the current distribution $\boldsymbol{I}$ and the vector potential $\boldsymbol{A}$:

$$\boldsymbol{A}(x,y,z) = \frac{\mu}{4\pi}\int_{C'} \boldsymbol{I}(x',y',z')\,\frac{e^{-jkR}}{R}\,dl'$$

where $(x,\ y,\ z)$ represent the observation point coordinates, $(x',\ y',\ z')$ represent the coordinates of the source, R is the distance from any point on the source to the observation point, and C' path is along the length of the source.

Since we have an infinitesimal current element kept at the origin $x' = y' = z' = 0$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = \sqrt{x^2+y^2+z^2} = r$$

- The vector potential due to a current element can be written as:

$$\boldsymbol{A}(x,y,z) = \hat{\boldsymbol{a}}_z \frac{\mu}{4\pi}(I_0 dl)\,\frac{e^{-jkr}}{r} = \hat{\boldsymbol{a}}_z \boldsymbol{A_z} \qquad \text{-------- (1.100)}$$

- Note that the vector potential has the same vector direction as the current element. In this case, the $\hat{\boldsymbol{a}}_z$ directed current element produces only the $\boldsymbol{A_z}$ component of the vector potential.

- The transformation between rectangular and spherical components is given, in matrix form:

$$\begin{bmatrix}\boldsymbol{A_r}\\ \boldsymbol{A_\theta}\\ \boldsymbol{A_\phi}\end{bmatrix} = \begin{bmatrix}\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta\\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta\\ -\sin\phi & \cos\phi & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{A_x}\\ \boldsymbol{A_y}\\ \boldsymbol{A_z}\end{bmatrix} \qquad \text{-------- (1.101)}$$

Substituting $\boldsymbol{A_x} = \boldsymbol{A_y} = 0$ in Eqn. (1.101):

$$\left.\begin{aligned}\boldsymbol{A_r} &= \boldsymbol{A_z}\cos\theta\\ \boldsymbol{A_\theta} &= -\boldsymbol{A_z}\sin\theta\\ \boldsymbol{A_\phi} &= 0\end{aligned}\right\} \qquad \text{-------- (1.102)}$$

Taking the curl of $\boldsymbol{A}$ in spherical coordinates;

$$\nabla\times\boldsymbol{A} = \frac{1}{r^2\sin\theta}\begin{bmatrix}\hat{\boldsymbol{a}}_r & r\hat{\boldsymbol{a}}_\theta & r\sin\theta\,\hat{\boldsymbol{a}}_\phi\\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi\\ \boldsymbol{A}_r & r\boldsymbol{A}_\theta & r\sin\theta\,\boldsymbol{A}_\phi\end{bmatrix}$$

$$\nabla\times\boldsymbol{A} = \frac{1}{r^2\sin\theta}\begin{bmatrix}\hat{\boldsymbol{a}}_r & r\hat{\boldsymbol{a}}_\theta & r\sin\theta\,\hat{\boldsymbol{a}}_\phi\\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi\\ \boldsymbol{A}_z\cos\theta & -r\boldsymbol{A}_z\sin\theta & 0\end{bmatrix}$$

$$\nabla\times\boldsymbol{A} = \frac{1}{r^2\sin\theta}\left[\begin{aligned}&\hat{\boldsymbol{a}}_r\left(\frac{\partial}{\partial\theta}(\boldsymbol{0}) - \frac{\partial}{\partial\phi}(-r\boldsymbol{A}_z\sin\theta)\right) - r\hat{\boldsymbol{a}}_\theta\left(\frac{\partial}{\partial r}(\boldsymbol{0}) - \frac{\partial}{\partial\phi}(\boldsymbol{A}_z\cos\theta)\right)\\ &\qquad + r\sin\theta\,\hat{\boldsymbol{a}}_\phi\left(\frac{\partial}{\partial r}(-r\boldsymbol{A}_z\sin\theta) - \frac{\partial}{\partial\theta}(\boldsymbol{A}_z\cos\theta)\right)\end{aligned}\right]$$

Since $\boldsymbol{A_z}$ is a function of r alone (from Eqn. 1.100), its derivative with respect to θ and ϕ are zero. Hence, the curl equation reduces to;

$$\nabla\times\boldsymbol{A} = \hat{\boldsymbol{a}}_\phi \frac{1}{r}\left[\frac{\partial}{\partial r}(-r\boldsymbol{A_z}\sin\theta) - \frac{\partial}{\partial\theta}(\boldsymbol{A_z}\cos\theta)\right] \qquad \text{-------- (1.103)}$$

Let ; $\quad \boldsymbol{A_z} = \dfrac{\mu(I_0 dl)}{4\pi}\dfrac{e^{-jkr}}{r}$

$$\nabla \times \boldsymbol{A} = \hat{\boldsymbol{a}}_\phi \; -\frac{1}{r}\left[\frac{\partial}{\partial r}\left(-r\left(\frac{\mu(I_0 dl)}{4\pi}\frac{e^{-jkr}}{r}\right)\sin\theta\right) - \frac{\partial}{\partial\theta}\left(\left(\frac{\mu(I_0 dl)}{4\pi}\frac{e^{-jkr}}{r}\right)\cos\theta\right)\right]$$

$$\nabla \times \boldsymbol{A} = \hat{\boldsymbol{a}}_\phi \frac{1}{r}\left[\frac{\mu(I_0 dl)}{4\pi}\frac{e^{-jkr}}{r}\sin\theta\,(jkr+1)\right]$$

$$\nabla \times \boldsymbol{A} = \hat{\boldsymbol{a}}_\phi \frac{1}{r}\boldsymbol{A}_z \sin\theta\,(jkr+1) \qquad \text{-------- (1.104)}$$

Eqn. (1.104) in (1.98):

$$\boldsymbol{H} = \hat{\boldsymbol{a}}_\phi \frac{1}{\mu r}\boldsymbol{A}_z \sin\theta\,(jkr+1) \qquad \text{-------- (1.105)}$$

$$\boldsymbol{H}_r\hat{\boldsymbol{a}}_r + \boldsymbol{H}_\theta\hat{\boldsymbol{a}}_\theta + \boldsymbol{H}_\phi\hat{\boldsymbol{a}}_\phi = \hat{\boldsymbol{a}}_\phi \frac{1}{\mu r}\boldsymbol{A}_z \sin\theta\,(jkr+1)$$

- The expressions for the components of the $\boldsymbol{H}$ field of a Hertzian dipole in spherical coordinates as:

$$\boldsymbol{H}_r = 0$$
$$\boldsymbol{H}_\theta = 0$$
$$\boldsymbol{H}_\phi = jk\frac{I_0 dl \sin\theta}{4\pi}\frac{e^{-jkr}}{r}\left[1+\frac{1}{jkr}\right] \qquad \text{-------- (1.106)}$$

Consider Eqn. (1.99):

$$\boldsymbol{E} = \frac{1}{j\omega\epsilon}(\nabla \times \boldsymbol{H}) = \frac{1}{j\omega\epsilon}\left(\frac{1}{r^2\sin\theta}\begin{bmatrix}\hat{\boldsymbol{a}}_r & r\hat{\boldsymbol{a}}_\theta & r\sin\theta\,\hat{\boldsymbol{a}}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 0 & 0 & r\sin\theta\,\boldsymbol{H}_\phi\end{bmatrix}\right)$$

$$\boldsymbol{E} = \frac{1}{j\omega\epsilon}\frac{1}{r^2\sin\theta}\left[\hat{\boldsymbol{a}}_r\frac{\partial}{\partial\theta}(r\sin\theta\;\boldsymbol{H}_\phi) - r\hat{\boldsymbol{a}}_\theta\frac{\partial}{\partial r}(r\sin\theta\;\boldsymbol{H}_\phi)\right]$$

$$\boldsymbol{E} = \frac{1}{j\omega\epsilon}\frac{1}{r^2\sin\theta}\left\{\begin{array}{l}\hat{\boldsymbol{a}}_r\frac{\partial}{\partial\theta}\left(r\sin\theta\left(jk\frac{I_0 dl\,\sin\theta}{4\pi}\frac{e^{-jkr}}{r}\left[1+\frac{1}{jkr}\right]\right)\right)\\ -r\hat{\boldsymbol{a}}_\theta\frac{\partial}{\partial r}\left(r\sin\theta\left(jk\frac{I_0 dl\,\sin\theta}{4\pi}\frac{e^{-jkr}}{r}\left[1+\frac{1}{jkr}\right]\right)\right)\end{array}\right\}$$

$$\boldsymbol{E} = \frac{1}{j\omega\epsilon}\frac{1}{r^2\sin\theta}\frac{jkI_0 dl}{4\pi}\left\{\begin{array}{l}\hat{\boldsymbol{a}}_r e^{-jkr}\left[1+\frac{1}{jkr}\right]\frac{\partial}{\partial\theta}(\sin^2\theta)\\ -r\hat{\boldsymbol{a}}_\theta(\sin^2\theta)\frac{\partial}{\partial r}\left(e^{-jkr}\left[1+\frac{1}{jkr}\right]\right)\end{array}\right\}$$

$$\boldsymbol{E} = \frac{k}{\omega\epsilon}\frac{1}{r^2\sin\theta}\frac{I_0 dl}{4\pi}\left\{\begin{array}{l}\hat{\boldsymbol{a}}_r e^{-jkr}\left[1+\frac{1}{jkr}\right]2\sin\theta\cos\theta\\ -r\hat{\boldsymbol{a}}_\theta(\sin^2\theta)\left(e^{-jkr}\left[-\frac{1}{jkr^2}\right]+\left[1+\frac{1}{jkr}\right]e^{-jkr}(-jk)\right)\end{array}\right\}$$

$$E_r \hat{a}_r + E_\theta \hat{a}_\theta + E_\phi \hat{a}_\phi = \eta \frac{1}{r^2 \sin\theta} \frac{I_0 dl}{4\pi} e^{-jkr} \sin\theta \left\{ \hat{a}_r (2\cos\theta)\left[1 + \frac{1}{jkr}\right] + \hat{a}_\theta (\sin\theta)(jk)\left(\left[1 + \frac{1}{jkr} + \frac{1}{jk(jk)r^2}\right]\right) \right\}$$

- The expressions for the components of the $\boldsymbol{E}$ field of a Hertzian dipole in spherical coordinates as:

$$\boldsymbol{E_r} = \eta \frac{I_0 dl \cos\theta}{2\pi r} \frac{e^{-jkr}}{r}\left[1 + \frac{1}{jkr}\right]$$
$$\boldsymbol{E_\theta} = j\eta \frac{k I_0 dl \sin\theta}{4\pi} \frac{e^{-jkr}}{r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right] \quad \text{-------- (1.107)}$$
$$\boldsymbol{E_\phi} = 0$$

where $\eta = k/(\omega\epsilon)$ is the intrinsic impedance of the medium.

Special case – Hertzian dipole:

- The expressions for the field components $\boldsymbol{E}$ and $\boldsymbol{H}$ of a Hertzian dipole in spherical coordinates as;

$$\boldsymbol{E_r} = \eta \frac{I_0 dl \cos\theta}{2\pi} e^{-jkr}\left[\frac{1}{r^2} + \frac{1}{jkr^3}\right]$$
$$\boldsymbol{E_\theta} = j\eta \frac{k I_0 dl \sin\theta}{4\pi} e^{-jkr}\left[\frac{1}{r} + \frac{1}{jkr^2} - \frac{1}{k^2 r^3}\right]$$
$$\boldsymbol{E_\phi} = \boldsymbol{H_r} = \boldsymbol{H_\theta} = 0$$
$$\boldsymbol{H_\phi} = jk \frac{I_0 dl \sin\theta}{4\pi} e^{-jkr}\left[\frac{1}{r} + \frac{1}{jkr^2}\right]$$

- The z- directed current element kept at the origin has only the $\boldsymbol{H_\phi}$, $\boldsymbol{E_r}$ and $\boldsymbol{E_\theta}$ components and, further, the field components that decay as $1/r$, $1/r^2$ and $1/r^3$, away from the current element.

$1/r$ term in $\boldsymbol{E}$ and $\boldsymbol{H}$ fields is called Radiation Field, $1/r^2$ term is called Induction Field and $1/r^3$ term is called Electrostatic Field.

⇨ Near field region: ($kr \ll 1$)

$$\boldsymbol{E_r} = \eta \frac{I_0 dl \ \cos\theta}{2\pi} e^{-jkr}\left[\frac{1}{jkr^3}\right] \cong -j\eta \frac{I_0 dl \ \cos\theta}{2\pi k r^3} e^{-jkr}$$
$$\boldsymbol{E_\theta} = j\eta \frac{k I_0 dl \ \sin\theta}{4\pi} e^{-jkr}\left[-\frac{1}{k^2 r^3}\right] \cong -j\eta \frac{I_0 dl \sin\theta}{4\pi k r^3} e^{-jkr}$$
$$\boldsymbol{E_\phi} = \boldsymbol{H_r} = \boldsymbol{H_\theta} = 0$$
$$\boldsymbol{H_\phi} = jk \frac{I_0 dl \ \sin\theta}{4\pi} e^{-jkr}\left[\frac{1}{jkr^2}\right] \cong \frac{I_0 dl \ \sin\theta}{4\pi r^2} e^{-jkr}$$

⇨ **Intermediate-field region:** $(kr > 1)$

$$\boldsymbol{E_r} = \eta \frac{I_0 dl \ \cos\theta}{2\pi} e^{-jkr} [\frac{1}{r^2}] \cong \eta \frac{I_0 dl \ \cos\theta}{2\pi r^2} e^{-jkr}$$

$$\boldsymbol{E_\theta} = j\eta \frac{kI_0 dl \ \sin\theta}{4\pi} e^{-jkr} [\frac{1}{r}] \cong j\eta \frac{kI_0 dl \sin\theta}{4\pi r} e^{-jkr}$$

$$\boldsymbol{E_\phi} = \boldsymbol{H_r} = \boldsymbol{H_\theta} = 0$$

$$\boldsymbol{H_\phi} = jk \frac{I_0 dl \ \sin\theta}{4\pi} e^{-jkr} [\frac{1}{r}] \cong j \frac{kI_0 dl \sin\theta}{4\pi r} e^{-jkr}$$

⇨ **Far-field region:** $(kr \gg 1)$

$$\boldsymbol{E_\theta} = j\eta \frac{kI_0 dl \ \sin\theta}{4\pi} e^{-jkr} [\frac{1}{r}] \cong j\eta \frac{kI_0 dl \sin\theta}{4\pi r} e^{-jkr}$$

$$\boldsymbol{E_r} = \boldsymbol{E_\phi} = \boldsymbol{H_r} = \boldsymbol{H_\theta} = 0$$

$$\boldsymbol{H_\phi} = jk \frac{I_0 dl \ \sin\theta}{4\pi} e^{-jkr} [\frac{1}{r}] \cong j \frac{kI_0 dl \sin\theta}{4\pi r} e^{-jkr}$$

- In the far-field, $\boldsymbol{E_\theta}$ and $\boldsymbol{H_\phi}$ are perpendicular to each other and transverse to the direction of propagation. The ratio of the two field components is same as the intrinsic impedance, η, of the medium

$$\frac{\boldsymbol{E_\theta}}{\boldsymbol{H_\phi}} = \eta \qquad \text{-------- (1.108)}$$

⇨ <u>**Power radiated and Radiation resistance:**</u>

- The time average power density (average Poynting vector) is given by;

$$\frac{1}{2} \qquad \frac{1}{2} \qquad \frac{1}{2\eta} \qquad \text{-------- (1.109)}$$

$$\frac{1}{2\eta} \quad \frac{kI_0 dl \sin\theta}{4\pi r} \quad ^{\mathbf{2}} \qquad \frac{1}{2} \quad \frac{kI_0 dl}{4\pi} \ ^{\mathbf{2}} \frac{\sin^2\theta}{r^2}$$

- The radiation intensity $U(\theta, \phi)$ is given by;

$$U(\theta,\phi) = r^2 \boldsymbol{W_{rad}} = r^2 \{\frac{1}{2}\eta \left|\frac{kI_0 dl}{4\pi}\right|^{\mathbf{2}} \frac{\sin^2\theta}{r^2}\} = \frac{1}{2}\eta \left|\frac{kI_0 dl}{4\pi}\right|^{\mathbf{2}} \sin^2\theta \qquad \text{-------- (1.110)}$$

- The average power radiated by an antenna (radiated power) can be written as ;

$$P_{rad} = \oiint_{\Omega} U(\theta,\phi)\, d\Omega = \int_0^{2\pi} \int_0^{\pi} U(\theta,\phi) \sin\theta \, d\theta \, d\phi \qquad \text{-------- (1.111)}$$

$$= \int_0^{2\pi} \int_0^{\pi} (\frac{1}{2}\eta \left|\frac{kI_0 dl}{4\pi}\right|^{\mathbf{2}} \sin^2\theta) \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{2}\eta \left|\frac{kI_0 dl}{4\pi}\right|^{\mathbf{2}} \int_0^{2\pi} \int_0^{\pi} \sin^3\theta \, d\theta \, d\phi$$

Substitute $\eta = 120\pi$ (for free space medium) $k = 2\pi/\lambda$;

$$P_{rad} = \frac{I_0^2}{2}\left[80\pi^2\left(\frac{dl}{\lambda}\right)^2\right]$$

$$\int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, d\theta \, d\phi = (2\pi)\left(\frac{4}{3}\right)$$

- The total power radiated by an antenna is given by;

$$P_{rad} = \frac{1}{2}|I_0|^2 R_r \qquad \text{-------- (1.112)}$$

- Radiation resistance is given by;

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 \qquad \text{-------- (1.113)}$$

⇨ Directive Gain (D) and Directivity(D_0):

- Directive Gain is given by ;

$$D = \frac{4\pi U}{P_{rad}} = \frac{4\pi\left[\frac{1}{2}\eta\left|\frac{kI_0 dl}{4\pi}\right|^2 \sin^2\theta\right]}{\frac{I_0^2}{2}\left[80\pi^2\left(\frac{dl}{\lambda}\right)^2\right]} = 1.5\sin^2\theta \qquad \text{-------- (1.114)}$$

- Directivity (maximum directivity, D_{max} or D_0) , maximum value occurs at $\theta = \pi/2$;

$$D_{max} = D_0 = 1.5 \qquad \text{-------- (1.115)}$$

- The maximum effective aperture is given by ;

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = 0.119\,\lambda^2 \qquad \text{-------- (1.116)}$$

HALF WAVE LENGTH DIPOLE: (Hertz Antenna or Half wave doublet)

- Half wave dipole is a linear antenna whose length is $\lambda/2$ and the current distribution is sinusoidal.

- It is a vertical radiator fed in the center. It produces maximum radiation in the plane normal to the axis. For a center-fed dipole of length l , symmetrically placed about the origin with its axis along the z-axis as shown in Fig. 1-20 (a).

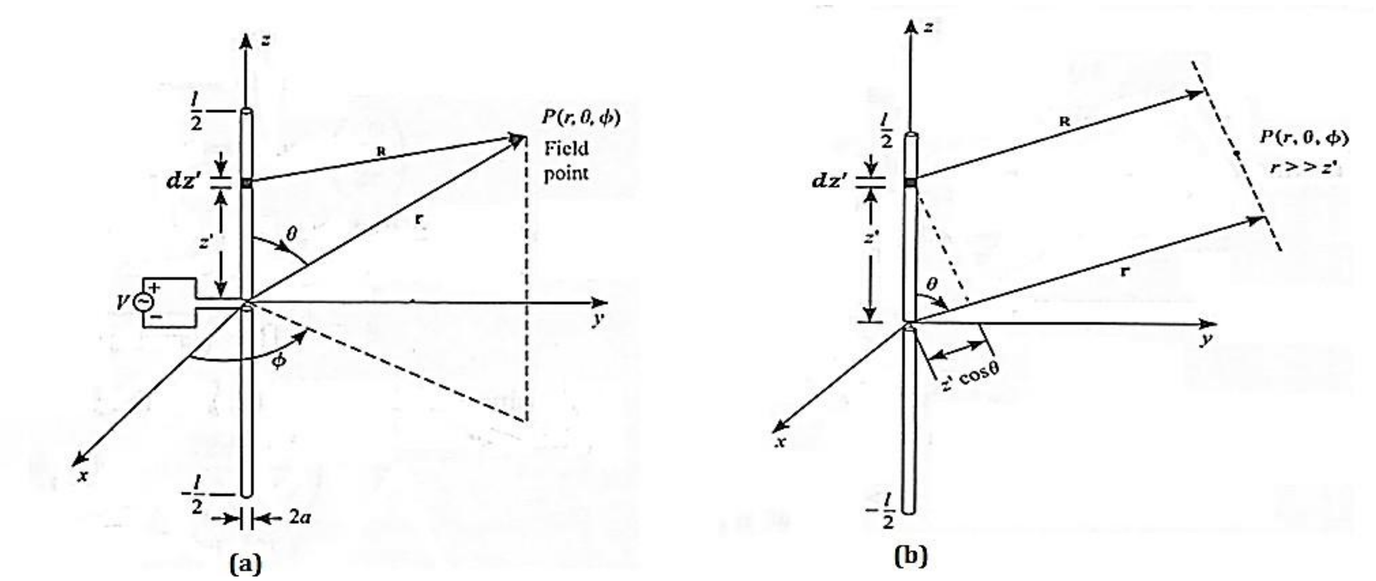


Fig. 1-20 (a) Geometry of a Have wavelength dipole (b) The far-field approximation

- As the length of the dipole approaches a significant fraction of the wavelength, it is found that the current distribution is closer to a sinusoidal distribution (Fig. 1-21) than a triangular distribution.
- The half wave dipole consists two legs each of length $l/2$. The physical length of the half wave dipole at the frequency of operation is $\boldsymbol{\lambda/2}$ in free-space.

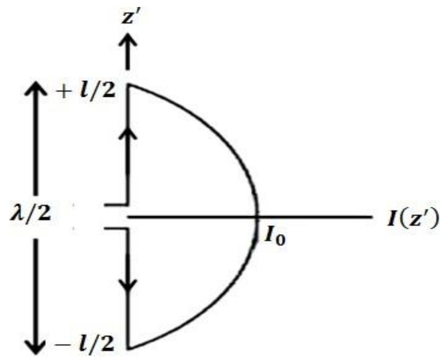


Fig. 1-21 Current distributionon a half-wave dipole excited at the centre

- Thus, the current on the dipole is given by ;

$$I_z(z') = \begin{cases} \hat{a}_z I_0 \sin\left[k\left(\frac{l}{2} - z'\right)\right] & 0 \le z' \le \frac{l}{2} \\ \hat{a}_z I_0 \sin\left[k\left(\frac{l}{2} + z'\right)\right] & -\frac{l}{2} \le z' \le 0 \end{cases} \quad \text{-------- (1.117)}$$

- Since the current is z-directed, the magnetic vector potential, $\boldsymbol{A}$, has only a z–component (i.e., $\boldsymbol{A_z}$). Therefore the vector potential is given by;
- Consider the relationship between the current distribution $\boldsymbol{I}$ and the vector potential $\boldsymbol{A}$:

$$\boldsymbol{A}(x,y,z) = \frac{\mu}{4\pi} \int_{C'} \boldsymbol{I}(x',y',z') \frac{e^{-jkR}}{R} dl' \quad \text{-------- (1.118)}$$

where R is the distance from of point $\boldsymbol{P}$ from the current element dz', and assume that $dl' = dz'$.

$$\boldsymbol{A}(x,y,z) = \hat{a}_z \frac{\mu}{4\pi} \left(\int_{-l/2}^{0} I_0 \sin\left[k\left(\frac{l}{2} + z'\right)\right] + \int_{0}^{l/2} I_0 \sin\left[k\left(\frac{l}{2} - z'\right)\right] \right) \frac{e^{-jkR}}{R} dz' \quad \text{-------- (1.119)}$$

- Geometrically, the far-field approximation implies that the vectors R and r are parallel to each other and a path difference of $z' \cos\theta$ exists between the two (Fig. 1-20 (b)).

$$\text{Far – field approximations} \quad (r \gg z'): \quad \begin{cases} R \cong r - z'\cos\theta & \text{for phase terms} \\ R \cong r & \text{for amplitude terms} \end{cases}$$

- Substitute the above approximations in Eqn. (1.119) ;

$$\boldsymbol{A}(x,y,z) = \hat{\boldsymbol{a}}_z \frac{\mu}{4\pi} I_0 \left(\int_{-l/2}^{0} \sin\left[k\left(\frac{l}{2}+z'\right)\right] + \int_{0}^{l/2} \sin\left[k\left(\frac{l}{2}-z'\right)\right]\right) \frac{e^{-jk(r-z'\cos\theta)}}{r} dz'$$

$$= \hat{\boldsymbol{a}}_z \frac{\mu}{4\pi} I_0 \frac{e^{-jkr}}{r} \left(\int_{-l/2}^{0} \sin\left[k\left(\frac{l}{2}+z'\right)\right] + \int_{0}^{l/2} \sin\left[k\left(\frac{l}{2}-z'\right)\right]\right) e^{jkz'\cos\theta} dz'$$

$$= \hat{\boldsymbol{a}}_z \frac{\mu}{4\pi} I_0 \frac{e^{-jkr}}{r} \left(\int_{-l/2}^{0} \sin\left[k\left(\frac{l}{2}+z'\right)\right] e^{jkz'\cos\theta} + \int_{0}^{l/2} \sin\left[k\left(\frac{l}{2}-z'\right)\right] e^{jkz'\cos\theta} \right) dz' \quad \text{-------- (1.120)}$$

Note: For half wave dipole $l = \lambda/2$ and $k = 2\pi/\lambda$

$$\sin\left[k\left(\frac{l}{2}-z'\right)\right] = \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}-z'\right)\right] = \cos kz' \; ; \quad \sin\left[k\left(\frac{l}{2}+z'\right)\right] = \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}+z'\right)\right] = \cos kz'$$

$$\boldsymbol{A}(x,y,z) = \hat{\boldsymbol{a}}_z \frac{\mu}{4\pi} I_0 \frac{e^{-jkr}}{r} \left(\int_{-l/2}^{0} \cos kz'\, e^{jkz'\cos\theta} + \int_{0}^{l/2} \cos kz'\, e^{jkz'\cos\theta} \right) dz'$$

$$= \hat{\boldsymbol{a}}_z \frac{\mu}{4\pi} I_0 \frac{e^{-jkr}}{r} \left(\int_{0}^{l/2} \cos(-kz')\, e^{-jkz'\cos\theta} + \int_{0}^{l/2} \cos kz'\, e^{jkz'\cos\theta} \right) dz'$$

$$\boldsymbol{A}(x,y,z) = \hat{\boldsymbol{a}}_z \frac{\mu}{4\pi} I_0 \frac{e^{-jkr}}{r} \left(\int_{0}^{l/2} \cos kz' \left(e^{-jkz'\cos\theta} + e^{jkz'\cos\theta}\right) \right) dz'$$

$$= \hat{\boldsymbol{a}}_z \frac{\mu}{4\pi} I_0 \frac{e^{-jkr}}{r} \left(\int_{0}^{\lambda/4} \cos kz' \left[2\cos(kz'\cos\theta)\right] \right) dz'$$

$$= \hat{\boldsymbol{a}}_z \frac{\mu}{2\pi} I_0 \frac{e^{-jkr}}{r} \left(\int_{0}^{\lambda/4} \cos kz' \left[\cos(kz'\cos\theta)\right] \right) dz' \quad \text{-------- (1.121)}$$

- By evaluating the integrals in Eqn. (1.121) and simplifying , we get approximately

$$\int_{0}^{\frac{\lambda}{4}} \cos kz' \left[\cos(kz'\cos\theta)\right] dz' = \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta}\right]$$

- Thus the vector potential, $\boldsymbol{A}$, is given by ;

$$\boldsymbol{A}(x,y,z) = \hat{\boldsymbol{a}}_z \frac{\mu}{2\pi} I_0 \frac{e^{-jkr}}{r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta}\right] = \hat{\boldsymbol{a}}_z A_z \quad \text{-------- (1.122)}$$

- The transformation between rectangular and spherical components is given, in matrix form:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \qquad \text{-------- (1.123)}$$

Substituting $A_x = A_y = 0$ in Eqn. (1.123):

$$\left.\begin{aligned} A_r &= A_z\cos\theta \\ A_\theta &= -A_z\sin\theta \\ A_\phi &= 0 \end{aligned}\right\} \qquad \text{-------- (1.124)}$$

****

Note: In the far-field of the dipole the electric and magnetic field intensities are transverse to each other as well as to the direction of propagation. E_θ, H_ϕ, and the direction of propagation, $\hat{a}_r$ form a right handed system. The ratio of E_θ/H_ϕ is equal to the the intrinsic impedance of the medium. ($E_\theta/H_\phi = \eta$)

The expressions for electric and magnetic field intensities are related to the magnetic vector potential by the following equations ;

$$E = -j\omega A_t \qquad \text{and} \qquad H = \frac{-j\omega}{\eta}\hat{a}_r \times A_t \qquad \text{-------- (1.125)}$$

where A_t represents the transverse component of the magnetic vector potential given by;

$$A_t = \hat{a}_\theta A_\theta + \hat{a}_\phi A_\phi \qquad \text{-------- (1.126)}$$

These equations are valid only in the far-field region .

****

- In the far-field of the z-oriented dipole, the component of the magnetic vector potential transverse to the direction of propagation is A_θ.
- The transverse component of the magnetic vector potential ,A_t, is given by;

$$A_t = \hat{a}_\theta(-A_z\sin\theta) \qquad \text{(By Eqn. 1.124 } A_\theta = -A_z\sin\theta\text{)}$$

- The expression for magnetic field intensity H, can be computed using Eqn. (1.125) ;

$$H = \frac{-j\omega}{\eta}\hat{a}_\phi\left(-\left[\frac{-j\omega}{\eta} I_0 \left[\frac{\pi}{\ }\right]\right]\sin\theta\right) \qquad \text{-------- (1.127)}$$

$$\boxed{\frac{I_0\, e^{-jkr}\cos\left(\frac{\pi}{2}\cos\theta\right)}{\ }} \qquad \text{-------- (1.128)}$$

where $\eta = (\omega\mu)/k$ is the intrinsic impedance of the medium.

- The expression for electric field intensity $\boldsymbol{E}$, can be computed using the following relationship.

$$\frac{\boldsymbol{E_\theta}}{\boldsymbol{H_\phi}} = \eta$$

$$\boldsymbol{E_\theta} = \eta \boldsymbol{H_\phi} = \eta\left[= j \frac{I_0}{2\pi} \frac{e^{-jkr}}{r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]\right]$$

$$\boldsymbol{E_\theta} = j\eta \frac{I_0}{2\pi} \frac{e^{-jkr}}{r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right] \qquad \text{-------- (1.129)}$$

⇨ <u>Power radiated and Radiation resistance:</u>

- The time average power density(average Poynting vector) is given by;

$$\boldsymbol{W_{rad}} = \frac{1}{2} Re[\boldsymbol{E} \times \boldsymbol{H}^*] = \frac{1}{2}[\hat{a}_\theta \boldsymbol{E_\theta} \times \hat{a}_\phi \boldsymbol{H_\phi}^*] = \hat{a}_r \frac{1}{2\eta} |\boldsymbol{E_\theta}|^2 \qquad \text{-------- (1.130)}$$

$$\boldsymbol{W_{rad}} = \hat{a}_r \frac{1}{2\eta} \left| j\eta \frac{I_0}{2\pi} \frac{e^{-jkr}}{r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right] \right|^2 = \hat{a}_r \frac{\eta I_0^2}{8\pi^2 r^2} \left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \right]$$

- The radiation intensity $U(\theta, \phi)$ is given by;

$$U(\theta,\phi) = r^2 \boldsymbol{W_{rad}} = r^2 \left\{ \frac{\eta I_0^2}{8\pi^2 r^2} \left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \right] \right\} = \frac{\eta I_0^2}{8\pi^2} \left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \right] \qquad \text{-------- (1.131)}$$

- The average power radiated by an antenna (radiated power) can be written as ;

$$P_{rad} = \oiint_\Omega U(\theta,\phi)\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \sin\theta\, d\theta\, d\phi \qquad \text{-------- (1.132)}$$

$$= \int_0^{2\pi}\int_0^{\pi} \left(\frac{\eta I_0^2}{8\pi^2} \left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \right] \sin\theta \right) d\theta\, d\phi$$

$$= \frac{\eta I_0^2}{8\pi^2} \int_0^{2\pi}\int_0^{\pi} \left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right] d\theta\, d\phi$$

Substitute $\eta = 120\pi$ (for free space medium)

$$\int_0^{2\pi}\int_0^{\pi} \left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right] d\theta\, d\phi = (2\pi)(1.218)$$

$$P_{rad} = 36.54\, I_0^2$$

- The total power radiated by an antenna is given by;

$$P_{rad} = \frac{1}{2} |I_0|^2 R_{rad} \qquad \text{-------- (1.133)}$$

- Radiation resistance is given by;

$$R_{rad} = 73\ \Omega \qquad \text{-------- (1.134)}$$

⇨ <u>Directive Gain (D) , Directivity (D_0) and Effective aperture (A_{em})):</u>

- Directive Gain is given by ;

$$D = \frac{4\pi U}{P_{rad}} = \frac{4\pi \frac{\eta I_0^2}{8\pi^2} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]^2}{36.54\ I_0^2} = 1.643 \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]^2 \qquad \text{-------- (1.135)}$$

- Directivity (maximum directivity, D_{max} or D_0) , maximum value occurs at $\theta = \pi/2$;

$$D_{max} = D_0 = 1.643 \qquad \text{-------- (1.136)}$$

- The maximum effective aperture is given by ;

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = 0.13\ \lambda^2 \qquad \text{-------- (1.137)}$$

<u>YAGI-UDA ANTENNA</u>:

- Yagi-Uda arrays or Yagi-Uda antennas are high gain antennas. The antenna was first invented by a Japanese Prof. S. Uda in early 1940's. Afterwards it was described in English by Prof. H. Yagi. Hence the antenna name Yagi-Uda antenna was given after Prof. S. Uda and Prof. H. Yagi.

- A basic Yagi-Uda antenna consists a driven element, one reflector and one or more directors. Basically it is an array of one driven element and one of more parasitic elements. The driven element is usually a folded dipole made of a metallic rod which is excited.

- A Yagi-Uda antenna uses both the reflector (R) and the director (D) elements in same antenna. The element at the back side of the driven element is the reflector. It is of the larger length compared with remaining elements. The element infront of the driven element is the director which is of lowest length in all the three elements.

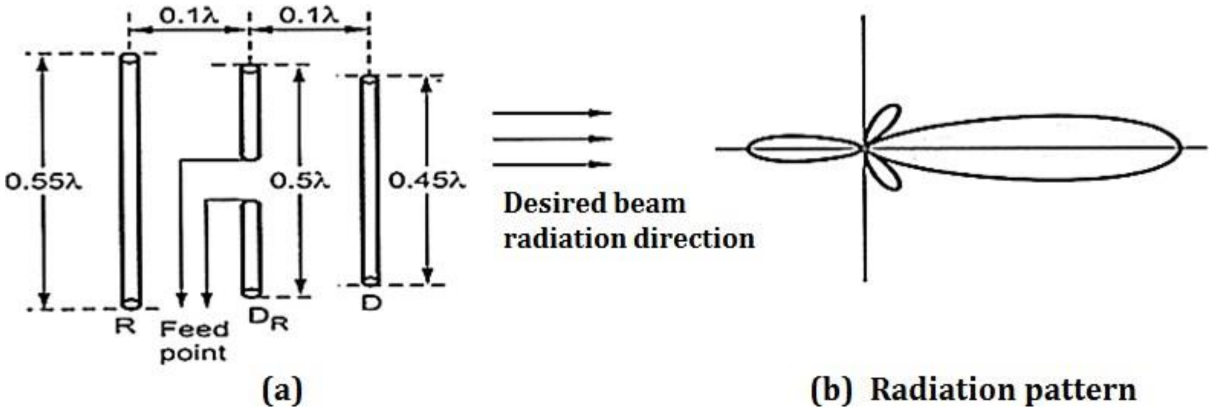


Fig. 1-22 Yagi-Uda antenna

- Directors and reflector are called parasitic elements. All the elements are placed parallel to each other and close to each other as shown in Fig. 1-22. The length of the folded dipole is about $\lambda/2$ and it is at resonance. Length of the director is less than $\lambda/2$ and length of the reflector is greater than $\lambda/2$.
- The parasitic element receive excitation through the induced e.m.f. as current flows in the driven element. The phase and amplitude of the currents through the parasitic elements mainly depends on the length of the elements and spacing between the elements. To vary reactance of any element, the dimensions of the elements are readjusted. Generally the spacing between the driven and the parasitic elements is kept nearly 0.1 λ to 0.15 λ.
- The lengths of the different elements can be obtained by using following formula:

$$Reflector\ length = \frac{152}{f_{MHz}}\ \ meter \qquad \text{-------- (1.138)}$$

$$Driver\ element\ length = \frac{143}{f_{MHz}}\ \ meter \qquad \text{-------- (1.139)}$$

$$Director\ length = \frac{137}{f_{MHz}}\ \ meter \qquad \text{-------- (1.140)}$$

- Let us consider the action of the Yagi-Uda antenna. The parasitic element is used either to direct or to reflect, the radiated energy forming compact directional antenna. If the parasitic element is greater than length $\lambda/2$, (i.e. reflector) then it is inductive in nature. Hence the phase of the current in such element i.e., in reflector lags the induced voltage.
- While if the parasitic element is less than the resonant length $\lambda/2$ (i.e. director), then it is capacitive in nature. Hence the current in director leads the induced voltage.
- The directors adds the fields of the driven element in the direction away from the driven element. If more than one directors are used, then each director will excite the next.
- Exactly opposite to this, properly spaced reflector adds the fields of the driven element in the direction towards driven element from the reflector. To increase the gain of the Yagi-Uda antenna, the number of directors is increased in the beam direction. To get good excitation, the elements are closely spaced.
- The driven element radiates from front to rear (i.e., from reflector to director). Part of this radiation induces currents in the parasitic elements which actually reradiate almost all radiations. With the proper lengths of the parasitic elements and the spacing between the elements, the backward radiation is cancelled and the radiated energy is added infront.
- When the spacing between the driven element and the parasitic element is reduced, the driven element gets loaded which reduces the input impedance at the terminals of the driven element. To overcome this the driven element used is the folded dipole which maintains the impedance at the input terminals.
- The Yagi-Uda antenna is the most widely used antenna for television signal reception. The gain of such antenna is very high and the radiation pattern is very much directive in one direction. The Yagi-Uda antenna along with its field radiation pattern is as shown in Fig. 1-23.

- The signal strength of the Yagi-Uda antenna can be increased by increasing number of directors in antenna.

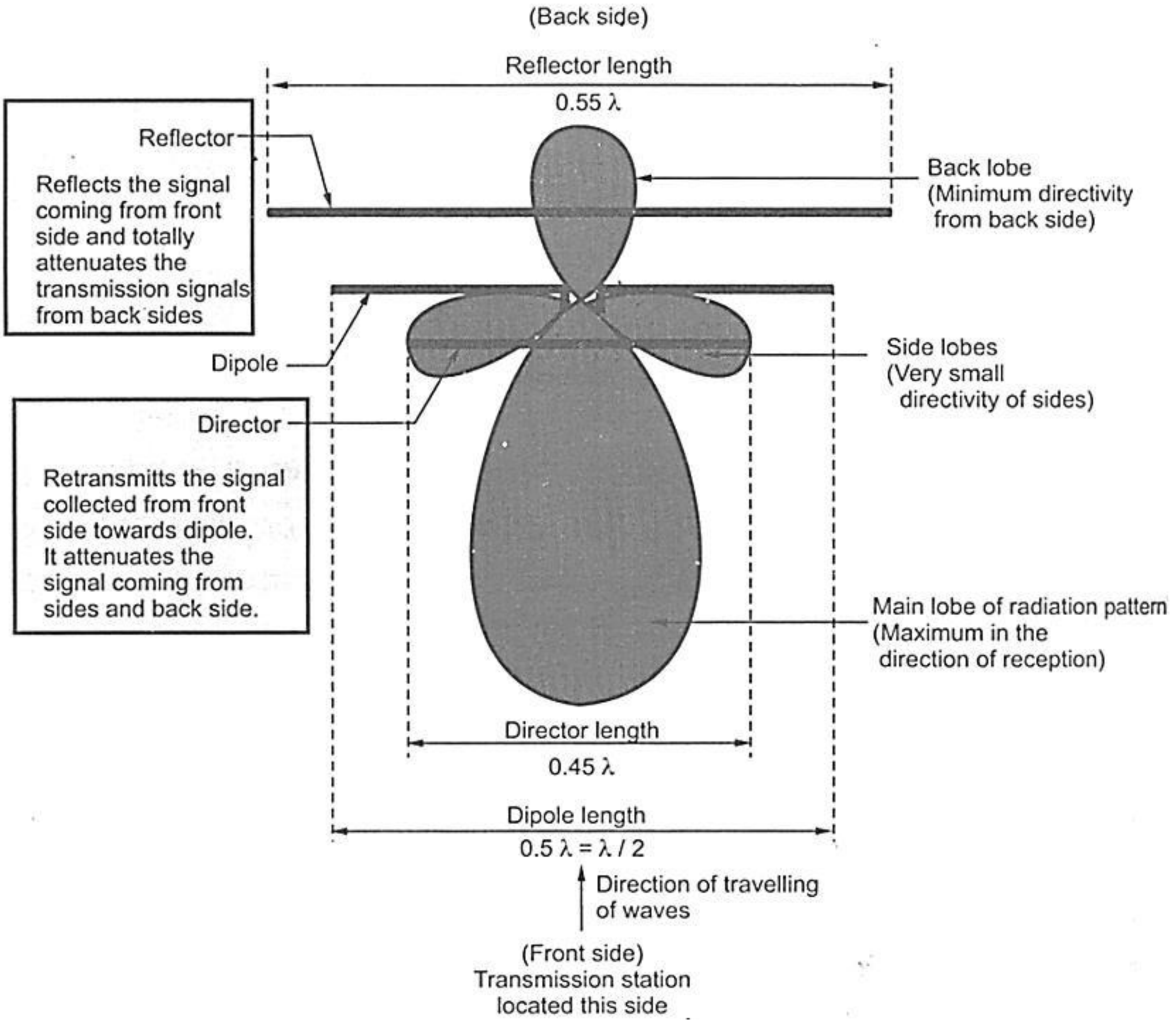


Fig. 1-23 Yagi antenna and its field radiation pattern

⇨ **Applications:**

- Yagi-Uda array is the most popular antenna for the reception of terrestrial television signals in the VHF band (30 MHz-300 MHz). The array for this application is constructed using aluminium pipes.
- The driven element is usually a folded dipole, which gives four times the impedance of a standard dipole. Thus, a two-wire balanced transmission line having a characteristic impedance of 300 Ω can be directly connected to the input terminals of the Yagi-Uda array.
- Yagi-Uda arrays have been used in the HF, VHF, UHF, and microwave frequency bands. In the HF band, the array is usually constructed using wires and at VHF and UHF frequencies, hollow pipes are used for the construction of Yagi-Uda arrays.
- At microwave frequencies, the array is constructed using either printed circuit board (PCB) technology or machined out of a metal sheet.

⇨ **Folded dipole:**

- To achieve good matching to practical coaxial lines with 50 Ω or 75 Ω characteristic impedances, the most widely used dipole is half wavelength $\lambda/2$, and which has an input impedance of $Z_{in} = 73 + j42.5$ and directivity of $D_{max} = 1.643$.
- In practice, there are other very common transmission lines whose characteristic impedance is much higher than 50 Ω or 75 Ω. For example, a "twin-lead" transmission line (usually two parallel wires) is widely used for TV applications and has a characteristic impedance of about 300 Ω.
- One simple geometry that can achieve this is a folded wire which forms a very thin rectangular loop as shown in Fig. 1-24.

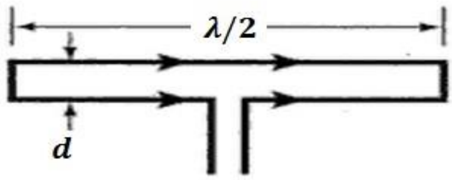


Fig. 1-24 2 Wire folded dipole antenna

- A folded dipole consists of two parallel $\lambda/2$ dipoles connected to each other at the ends. It is fed at the centre of one of the dipoles and the other dipole is shorted.
- The impedance of the folded dipole is four times greater than that of an isolated dipole of the same length, i.e., $\boldsymbol{Z_f = 4Z_d}$.
- **Advantages:**
  i. Very high input impedance
  ii. Inherent impedance transformation property
  iii. Wide bandwidth
  iv. Acts as reactance compensation network
- **Applications:** The folded dipoles with parasitic elements can be used for wideband transmissions such as TV signals. It is used in Yagi-uda antenna as a driven element.

Post – MCQ:

1. The polarization mismatch is defined as –
   (a) Polarization of the receiving antenna not matching with incoming wave
   (b) Polarization of the transmitting antenna not matching with incoming wave
   (c) Polarization of the receiver not matching with transmitter
   (d) All of the above

Ans: a

2. The balun is a device used to match –
   (a) Transmission line with dipole feed
   (b) Co-axial line with dipole feed
   (c) Transmitter with antenna load
   (d) None of the above

Ans:b

3. If the PLF parameter is unity, then –
   (a) Maximum power extracted from the incoming wave
   (b) Moderate power extracted from the incoming wave
   (c) No power extracted from the incoming wave
   (d) None of the above

Ans : a

4. The currents on the inner conductor and the inside of the outer conductor in a co-axial cable are equal in magnitude and opposite in direction, then it is said to be –
   (a) Balanced current mode
   (b) Unbalanced current mode
   (c) Normal current mode
   (d) None of the above

Ans:b

5. The resistance if connected in series with an antenna will consume the same power as actually radiated by the antenna is called as –
   (a) Lossy resistance
   (b) Antenna resistance
   (c) Radiation resistance
   (d) Any of the above

Ans: c

6. The radiation pattern of an isotropic antenna is –
   (a) Elliptical
   (b) Figure of eight
   (c) Spherical
   (d) Hyperbolic

Ans: c

7. The effective aperture indicates the amount of power –
   (a) Captured from the plane wave and delivered by the antenna
   (b) Reflected from the plane wave and delivered by the antenna
   (c) Refracted from the plane wave and delivered by the antenna
   (d) None of the above

Ans: a

8. The principle of reciprocity states that the receive and transmit antennas have-
   (a) Radically different radiation patterns
   (b) Identical radiation patterns
   (c) Slightly different radiation patterns
   (d) None of the above

Ans: b

9. A half wave folded dipole is an antenna with-
   (a) Half of wavelength and feed at the ends
   (b) Half of wavelength and feed at the centre
   (c) Half of wavelength and feed at anywhere
   (d) None of the above

Ans: b

10. The antenna noise temperature is –
   (a) Physical temperature of the antenna
   (b) Physical temperature of antenna and transmission line
   (c) Temperature at the receiver terminals in a thermal environment
   (d) Any of the above

Ans: c

11. Radiation intensity is defines as power per -
    (a) Unit area
    (b) Unit angle
    (c) Unit field
    (d) None of the above

Ans: b

12. The radiation intensity can be obtained by multiplying the radiation density by –
    (a) The distance
    (b) Square of the distance
    (c) Cube of the distance
    (d) None of the above

Ans: b

13. Directivity is defined as the ratio of the maximum radiation intensity to –
    (a) average radiation intensity
    (b) peak radiation intensity
    (c) unity radiation intensity
    (d) None of the above

Ans: a

14. The gain of an antenna is an actual quantity which is less than directivity (D) due to –
    (a) Radiating losses in the antenna
    (b) ohmic losses in the antenna
    (c) inductive losses in the antenna
    (d) All of the above

Ans: b

15. In an antenna system, the maximum power transfer takes place when the antenna is ___________to the source.
    (a) Perfectly matched
    (b) Inductively matched
    (c) Conjugate matched
    (d) None of the above

Ans: c

Conclusion:

At the end of the topic, students will be able –

1) To understand the basic concept of Antenna Fundamentals

2) To understand the different types of Antenna Parameters

3) To get exposure on Monopole and half wave dipole antennas

4) To know the EM wave equation of oscillating current elements.

References:

1. John D Kraus," Antennas for all Applications", 4th Edition, McGraw Hill, 2010.

2. Edward C. Jordan and Keith G. Balmain" Electromagnetic Waves and Radiating Systems" Prentice Hall of India, 2nd Edition 2011.

3. R.E. Collin," Antennas and Radio wave Propagation", McGraw Hill 1985.

4. Robert S. Elliott "Antenna Theory and Design" Wiley Student Edition, 2006.

Assignments:

1. Explain briefly the various antenna parameters with necessary equations.

2. Derive an expression for the radiation from an oscillating current element with diagram.

3. Derive an expression for the radiation from a half-wave dipole with diagram

4. Write short notes on – (i) Matching Baluns (ii) Antenna Temperature

5. Explain about Polarization mismatch in a brief manner

Subject Name: Antennas & Propagation

Topic Name: Antenna Arrays

(Unit – 2)

Prepared

By

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Associate Professor,

Department of ECE

SCSVMV

Syllabus / Antenna Arrays

1. Broad side arrays
2. End fire arrays
3. Collinear arrays

Example of Antenna Arrays:

Yagi Uda Arrays

Aim and Objective:

- To give insight of basic Knowledge of Antenna Arrays
- To give thorough understanding of the radiation characteristics of types of Antenna Arrays i.e Broad side array, End fire array and Collinear arrays.
- To impart knowledge about applications of Antenna Arrays

Pre – Test MCQ:

1. An antenna is a transitional structure between –
   (a) Free space and guiding element
   (b) Free space and free space
   (c) Guiding element and another guiding element
   (d) All of the above

Answer - a

2. The guiding device used for the antenna system –
   (a) Transmission line
   (b) Co-axial line
   (c) Waveguide
   (d) All of the above

Answer - d

3. If the radiation from an antenna is represented in terms of field strength, it is called –
   (a) Field pattern
   (b) Power pattern
   (c) Radiation pattern
   (d) None of the above

Answer - a

4. A major lobe is defined as the radiation lobe containing –
   (a) Direction of minimum radiation
   (b) Direction of maximum radiation
   (c) Direction of moderate radiation
   (d) None of the above

Answer - b

5. The First-null beam width (FNBW) is defined as the angular measurement between the directions –
   (a) radiating the maximum power
   (b) radiating half of the maximum power

(c) radiating no power
(d) None of the above

Answer - c

6. The half-power beam width (HPBW) is defined as the angular measurement between the directions –
(a) radiating the maximum power
(b) radiating half of the maximum power
(c) radiating no power
(d) None of the above

Answer - b

7. The x-z plane (elevation plane; ϕ = 0) is the principal –
(a) $\boldsymbol{E}$-plane
(b) H-plane
(c) Either E-plane or H-plane
(d) None of the above

Answer - a

8. An Omni directional antenna is a special antenna of type –
(a) Directional
(b) Non-directional
(c) Isotropic
(d) None of the above

Answer - a

9. The ratio of the main beam area to the total beam area is called –
(a) beam efficiency
(b) beam deficiency
(c) stray factor
(d) None of the above

Answer - a

10. Radiation density is defined as power per –
(a) Unit area
(b) Unit angle
(c) Unit field
(d) None of the above

Answer - a

11. Radiation intensity is defines as power per -
    (a) Unit area
    (b) Unit angle
    (c) Unit field
    (d) None of the above

Answer - b

12. The radiation intensity can be obtained by multiplying the radiation density by –
    (a) The distance
    (b) Square of the distance
    (c) Cube of the distance
    (d) None of the above

Answer - b

13. Directivity is defined as the ratio of the maximum radiation intensity to –
    (a) average radiation intensity
    (b) peak radiation intensity
    (c) unity radiation intensity
    (d) None of the above

Answer - a

14. The gain of an antenna is an actual quantity which is less than directivity (D) due to –
    (a) Radiating losses in the antenna
    (b) ohmic losses in the antenna
    (c) inductive losses in the antenna
    (d) All of the above

Answer - b

15. In an antenna system, the maximum power transfer takes place when the antenna is ___________to the source.
    (a) Perfectly matched
    (b) Inductively matched
    (c) Conjugate matched
    (d) None of the above

Answer - c

Pre-requisite

- Basic knowledge of Electromagnetic Fields and Wave guides.
- Basic Knowledge of Antenna Parameters and Radiations

UNIT II: ANTENNA ARRAYS

N element linear array, Pattern multiplication, Broadside and End fire array – Concept of Phased arrays, Adaptive array, Basic principle of antenna Synthesis-Binomial array, Yagi Arrays.

INTRODUCTION – ARRAY ANTENNAS:

- The radiation pattern of a single element is relatively wide, and each element provides low values of directivity (gain). In many applications, it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long distance communication. This can only be accomplished by increasing the electrical size of the antenna.

  Note: Higher directivity is the basic requirement in point-to-point communication, radars and space applications.

- Enlarging the dimensions of single elements often leads to more directive characteristics. Another way to enlarge the dimensions of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration*. This new antenna, formed by multielements, is referred to as an array*. Inmost cases, the elements of an array are identical. The individual elements of an array may be of any form (wires, apertures, etc.).
- Thus antenna array can be defined as the system of similar antennas directed to get required high directivity in the desired direction.
- The total field of the array is determined by the vector addition of the fields radiated by the individual elements. The individual element is generally called *element of an antenna array*. This assumes that the current in each element is the same as that of the isolated element (neglecting coupling).
- The antenna array is said to be *linear* if the elements of the antenna array are equally spaced along a straight line. The linear antenna array is said to be *uniform linear* array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line.
- In an array of identical elements, there are at least five controls that can be used to shapethe overall pattern of the antenna. These are:
  - the geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)
  - the relative displacement between the elements
  - the excitation amplitude of the individual elements
  - the excitation phase of the individual elements
  - the relative pattern of the individual elements

- Practically various forms of the antenna array are used as radiating systems. They are;
  i. Broadside Array (BSA)
  ii. End-Fire Array (EFA)
  iii. Collinear Array
  iv. Parasitic Array

⇨ **Broadside Array (BSA)**

- The broadside array is the array of antennas in which all the elements are placed parallel to each other and the direction of maximum radiation is always perpendicular to the plane consisting elements.A typical arrangement of a Broadside array is as shown in Fig. 3-1.
- A broadside array consists number of identical antennas placed parallel to each other along a straight line. This straight line is perpendicular to the axis of individual antenna. It is known as *axis of antenna array.* Thus each element is perpendicular to the axis of antenna array.
- All the individual antennas are spaced equally along the axis of antenna array. The spacing between any two elements is denoted by $'d'$. All the elements are fed with currents with equal magnitude and same phase. As the maximum radiation is directed in broadside direction i.e. perpendicular to the line of axis of array, the radiation pattern for the broadside array is bidirectional.
- Thus broadside array can be defined as the arrangement of antennas in which maximum radiation is in the direction perpendicular to the axis of array and plane containing the elements of array.

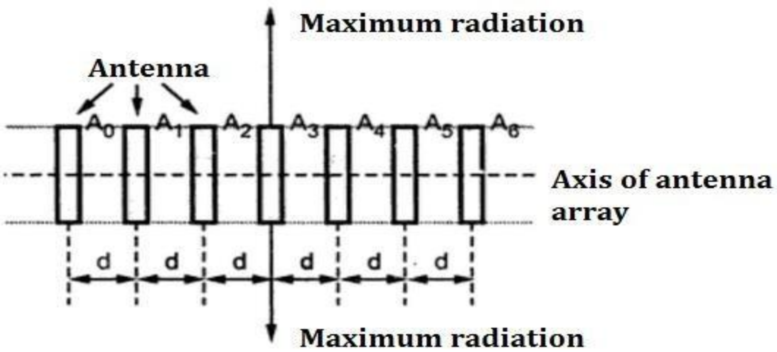


Fig. 3-1 Broadside array

⇨ **End-Fire Array (EFA)**

- The end fire array is very much similar to the broadside array from the point of view of arrangement. But the main difference is in the direction of maximum radiation. In broadside array, the direction of the maximum radiation is perpendicular to the axis of array; while in the end fire array, the direction of the maximum radiation is along the axis of array.

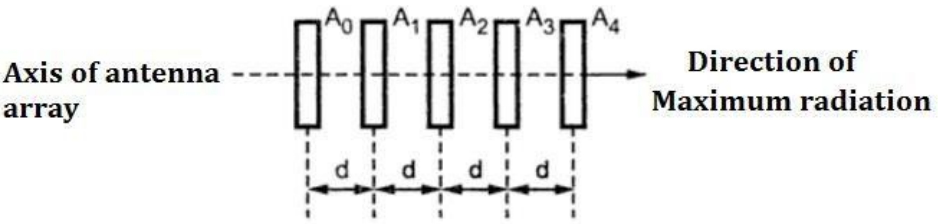


Fig. 3-2 End-fire array

- Thus in the end fire array number of identical antennas are spaced equally along a line. All the antennas are fed individually with currents of equal magnitudes but their phases vary progressively along the line to get entire arrangement unidirectional finally. i.e. maximum radiation along the axis of array as shown in Fig. 3-2.

- Thus end fire array can be defined as an array with direction of maximum radiation coincides with the direction of the axis of array to get unidirectional radiation.

⇨ **Collinear array**

- As the name indicates, in the collinear array, the antennas are arranged co-axially i.e. the antennas are arranged end to end along, a single line as shown in Fig. 3-3 (a) and (b).

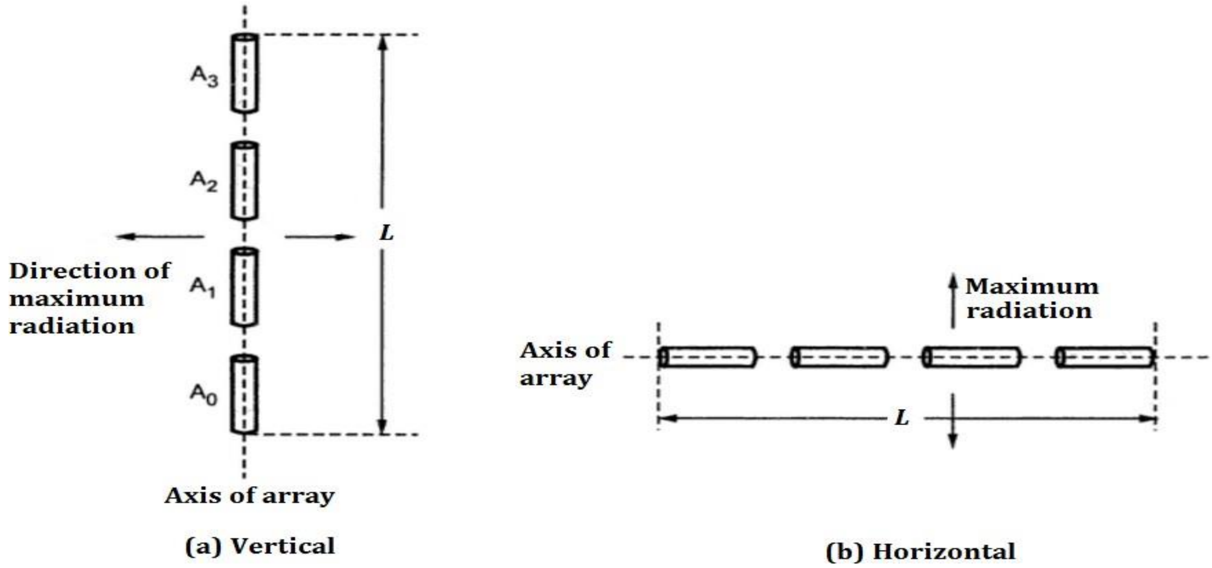


Fig. 3-3 Collinear array

- The individual elements in the collinear array are fed with currents equal in magnitude and phase. This condition is similar to the broadside array. In collinear array the direction of maximum radiation is perpendicular to the axis of array.
- So the radiation pattern of the collinear array and the broadside array is very much similar but the radiation pattern of the collinear array has circular symmetry with main lobe perpendicular everywhere to the principle axis. Thus the collinear array is also called omni directional array or broadcast array.
- The gain of the collinear array is maximum if the spacing between the elements is of the order of $0.3\ \lambda$ to $0.5\ \lambda$.

⇨ **Parasitic array**

- In order to overcome feeding problems of the antenna, sometimes, the elements of the array are fed through the radiation from the nearby element. The array of antennas in which the parasitic elements get the power through electromagnetic coupling with driven element which is in proximity with the parasitic element is known as parasitic array.
- The simplest form of the parasitic array consists one driven element and one parasitic element. In multielement parasitic array, there may be one or more driving elements and also one or more parasitic elements. So in general the multielement parasitic array is the array with at least one driven element and one or more parasitic elements.
- The common example of the parasitic array with linear half wave dipoles as elements of array is Yagi-Uda array or simply Yagi antenna.
- The amplitude and the phase of the current induced in the parasitic element depends on the spacing between the driven element and parasitic element. To make the radiation pattern

unidirectional, the relative phases of the currents are changed by adjusting the spacing between the elements. This is called tuning of array. For a spacing between the driven and parasitic element equal to $\lambda/4$ and phase difference of $\pi/2$ radian, unidirectional radiation pattern is obtained.

ARRAY OF POINT SOURCES

- The array of point sources is nothing but the array of an isotropic radiators occupying zero volume. For the greater number of point source in the array, the analysis of antenna array becomes complicated and time consuming. Also the simplest condition of number of point sources in the array is two. Then conveniently analysis is done by considering first two point sources, which are separated by distance d and having same polarization. The results obtained for only two point sources can be further extended for $'n'$ number of point sources in the array.
- Let us consider the array of two isotropic point sources, with a distance of separation $'d'$ between them. The polarization of two isotropic point sources is assumed to be the same. To derive different expressions following conditions can be applied to the antenna array ;
  1. Two point sources with currents of equal magnitudes and with same phase.
  2. Two point sources with currents of equal magnitude but with opposite phase.
  3. Two point sources with currents of unequal magnitudes and with any phase.

⇨ **Two Point Sources with Currents Equal in Magnitude and Phase:**

- Consider two point-sources 1 and 2 separated by distance $'d'$ and both the point sources are supplied with currents equal in magnitude and phase as shown in Fig. 3-4.
- Let point P far away from the array and the distance between point P and point sources 1 and 2 be r_1 and r_2 respectively.

  Assuming far-field observations ,

$$r_1 \approx r_2 \approx r \qquad \text{-------- (3.1)}$$

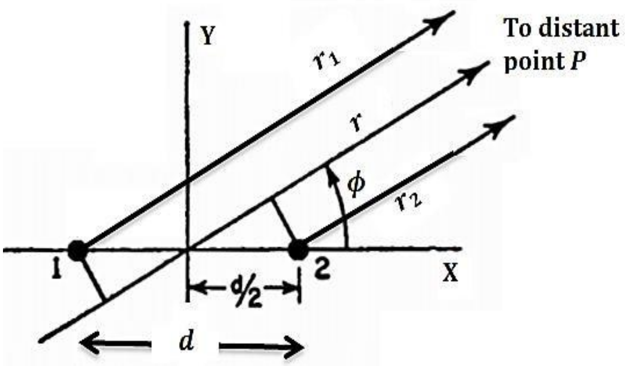


Fig. 3-4 Two element array

- The radiation from the point source 2 will reach earlier at point P than that from point source 1 because of the path difference. The extra distance is travelled by the radiated wave from point source 1 than that by the wave radiated from point source 2. Hence path difference is given by,

$$Path\ difference = \frac{d}{2}\cos\phi + \frac{d}{2}\cos\phi = d\cos\phi \qquad \text{-------- (3.2)}$$

- The path difference can be expressed in terms of wavelength as ;

$$Path\ difference = \frac{d}{\lambda}\cos\phi$$

Hence the phase difference $'\psi'$ is given by ;

$$Phase\ difference\ \psi = 2\pi \times Path\ difference$$

$$k = \frac{2\pi}{\lambda}$$

$$\psi = 2\pi \times \frac{d}{\lambda}\cos\phi = \frac{2\pi}{\lambda} d\cos\phi$$

$$\psi = kd\cos\phi \qquad \text{-------- (3.3)}$$

- Let $E_1 = E_0 . e^{-j\frac{\psi}{2}}$ is field component due topoint source 1. Similarly, let $E_2 = E_0 . e^{j\frac{\psi}{2}}$ is field component due to point source 2. Therefore, the total far-field at a distant point P is ;

$$E_T = E_1 + E_2 = E_0 . e^{-j\frac{\psi}{2}} + E_0 . e^{j\frac{\psi}{2}}$$

$$E_T = E_0\left(e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right) = 2E_0\cos\frac{\psi}{2} \qquad \text{-------- (3.4)}$$

Note that the amplitude of both the field components is E_0 as currents are same and the point sources are identical.

Substituting value of ψ from Eqn. (3.3), we get,

$$E_T = 2E_0\cos\left[\frac{kd\cos\phi}{2}\right] \qquad \text{-------- (3.5)}$$

- Above equation represents total field intensity at point P, due to two point sources having currents of same amplitude and phase. The total amplitude of the field at point P is $2E_0$ while the phase shift is $(kd\cos\phi)/2$. By putting $2E_0 = 1$, then the pattern is said to be normalized.

⇨ **<u>Maxima direction:</u>**

- From Eqn. (3.5), the total field is maximum when $\cos\left[\frac{kd\cos\phi}{2}\right]$ is maximum. Maximum value of cosine function is ±1. Hence the condition for maxima is given by,

$$\cos\left[\frac{kd\cos\phi}{2}\right] = \pm 1 \qquad \text{-------- (3.6)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$k = \frac{2\pi}{\lambda}\ ;\ d = \frac{\lambda}{2}$$

$$\cos\left[\frac{\pi}{2}\cos\phi\right] = \pm 1$$

$$i.e,\quad \frac{\pi}{2}\cos\phi_{max} = \cos^{-1}(\pm 1) = \pm n\pi,\ where,\ n = 0,1,2, ...$$

If $n = 0$, then ; $\quad \frac{\pi}{2}\cos\phi_{max} = 0$

$$\cos\phi_{max} = 0 \quad ; \quad \phi_{max} = 90°\ \text{or}\ 270° \qquad \text{--------------- (3.7)}$$

⇨ **Minima direction:**

- From Eqn. (3.5), the total field is minimum when $\cos\left[\frac{kd\cos\phi}{2}\right]$ is minimum. Minimum value of cosine function is 0. Hence the condition for maxima is given by,

$$\cos\left[\frac{kd\cos\phi}{2}\right] = 0 \qquad \text{-------- (3.8)}$$

- Let spacing between the two point sources be $\lambda/2$, then;

$$\cos\left[\frac{\pi}{2}\cos\phi\right] = 0$$

$$i.e, \qquad \frac{\pi}{2}\cos\phi_{mi} = \cos^{-1}(0) = \pm(2n+1)\ \frac{\pi}{2}, where, n = 0,1,2, \ldots.$$

$$\text{If } n = 0, \text{then}; \qquad \frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$$

$$\cos\phi_{min} = \pm 1 \quad ; \quad \phi_{min} = 0° \text{ or } 180° \qquad \text{-------- (3.9)}$$

⇨ **Half power point directions:**

- When the power is half, the voltage or current is $\frac{1}{\sqrt{2}}$ times the maximum value. Hence the condition for half power point is given by,

$$\cos\left[\frac{kd\cos\phi}{2}\right] = \pm\frac{1}{\sqrt{2}} \qquad \text{-------- (3.10)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$\cos\left[\frac{\pi}{2}\cos\phi\right] = \pm\frac{1}{\sqrt{2}}$$

$$i.e, \qquad \frac{\pi}{2}\cos\phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = \pm\frac{\pi}{4}, where, n = 0,1,2, \ldots.$$

$$\text{If } n = 0, \text{then}; \qquad \frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\phi_{HPPD} = \pm\frac{1}{2} \quad ; \quad \phi_{HPPD} = \pm 60° \text{ or } \pm 120° \qquad \text{-------- (3.11)}$$

- The field pattern drawn with E_T against ϕ for $d = \lambda/2$, then the pattern is bidirectional as shown in Fig. 3-5. The field pattern obtained is bidirectional and it is a figure of eight (8). If this patterns is rotated by 360° about axis, it will represent three dimensional doughnut shaped space pattern.

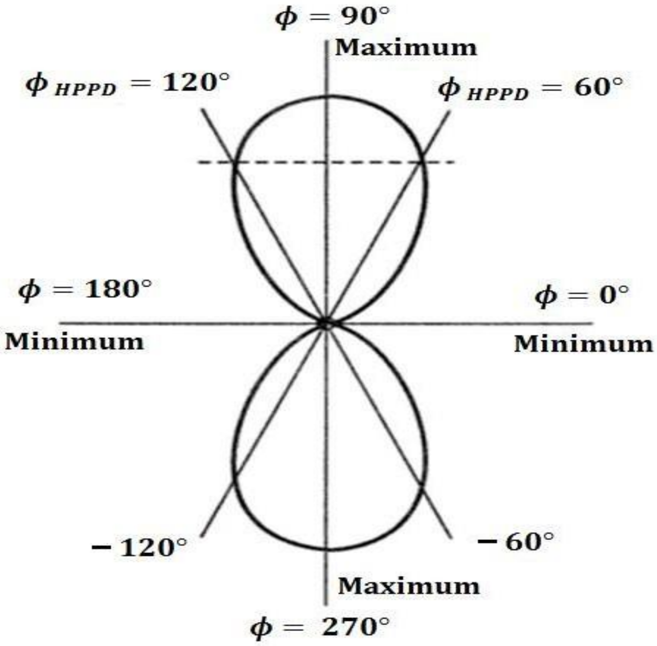


Fig. 3-5 Field pattern for two point source with $d = \lambda/2$ and fed with currents equal in magnitude and phase.

⇨ **<u>Two Point Sources with Currents Equal in Magnitude and opposite phase:</u>**

- Consider two point sources separated by distance $'d'$ and supplied with currents equalin magnitude but opposite phase. Consider Fig.3-4, all theconditions are exactly same except the phase of the currents is opposite i.e. 180°. Withthis condition, the total field at far point P is given by,

$$E_T = (-E_1) + E_2 \qquad \text{-------- (3.12)}$$

- Assuming equal magnitudes of currents, the fields at point P due to the point sources 1 and 2 can be written as ; $E_1 = E_0 . e^{-j\frac{\psi}{2}}$

$$E_2 = E_0 . e^{j\frac{\psi}{2}}$$

- Therefore, the total far-field at a distant point P is ;

$$E_T = (-E_1) + E_2 = -E_0 . e^{-j\frac{\psi}{2}} + E_0 . e^{j\frac{\psi}{2}}$$

$$E_T = E_0 \left(e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}\right) = j(2E_0) \sin\frac{\psi}{2} \qquad \text{-------- (3.13)}$$

- Now as the condition for two point sources with currents in phase and out of phaseis exactly same, the phase angle can be written as ;

$$\psi = kd \cos\phi \qquad \text{-------- (3.14)}$$

Substituting value of ψ in Eqn. (2.93), we get,

$$E_T = j(2E_0) \sin\left[\frac{kd\cos\phi}{2}\right] \qquad \text{-------- (3.15)}$$

By putting $(2E_0) = 1$, then the pattern is said to be normalized.

⇨ **Maxima direction:**

- From Eqn. (3.15), the total field is maximum when $\sin[\frac{kd\cos\phi}{2}]$ is maximum. Maximum value of sine function is ±1. Hence the condition for maxima is given by,

$$\sin[\frac{kd\cos\phi}{2}] = \pm 1 \quad \text{-------- (3.16)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$k = \frac{2\pi}{\lambda} \;;\; d = \frac{\lambda}{2}$$

$$\sin[\frac{\pi}{2}\cos\phi] = \pm 1$$

$$i.e, \quad \frac{\pi}{2}\cos\phi_{max} = \sin^{-1}(\pm 1) = \pm(2n+1)\frac{\pi}{2}, where, n = 0,1,2, ...$$

If $n = 0$, then ; $\frac{\pi}{2}\cos\phi_{mi} = \pm\frac{\pi}{2}$

$$\cos\phi_{max} = \pm 1 \quad ; \quad \phi_{ma} = 0° \text{or } 180° \text{--------------- (3.17)}$$

⇨ **Minima direction:**

- From Eqn. (3.15), the total field is minimum when $\sin[\frac{kd\cos\phi}{2}]$ is minimum. Minimum value of sine function is 0.Hence the condition for maxima is given by,

$$\sin[\frac{kd\cos\phi}{2}] = 0 \quad \text{-------- (3.18)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$\sin[\frac{\pi}{2}\cos\phi] = 0$$

$$i.e, \quad \frac{\pi}{2}\cos\phi_{min} = \sin^{-1}(0) = \pm n\pi, where, n = 0,1,2, ...$$

If $n = 0$, then ; $\frac{\pi}{2}\cos\phi_{min} = 0$

$$\cos\phi_{min} = 0 \quad ; \quad \phi_{min} = 90° \text{ or } 270° \quad \text{-------- (3.19)}$$

⇨ **Half power point directions:**

- When the power is half, the voltage or current is $\frac{1}{\sqrt{2}}$ times the maximum value. Hence the condition for half power point is given by,

$$\sin[\frac{kd\cos\phi}{2}] = \pm\frac{1}{\sqrt{2}} \quad \text{-------- (3.20)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$\sin[\frac{\pi}{2}\cos\phi] = \pm\frac{1}{\sqrt{2}}$$

$$i.e, \quad \frac{\pi}{2}\cos\phi_{HPPD} = \sin^{-1}(\pm\frac{1}{\sqrt{2}}) = \pm\frac{\pi}{4}, where, n = 0,1,2, ...$$

If $n = 0$, then ; $\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$

$$\cos\phi_{HPPD} = \pm\frac{1}{2}; \qquad \phi_{HPPD} = \pm 60° \text{ or } \pm 120° \qquad \text{-------- (3.21)}$$

- Thus from the conditions of maxima, minima and half power points, the field pattern can be drawn with E_T against ϕ for $d = \lambda/2$ as shown in Fig. 3-6.

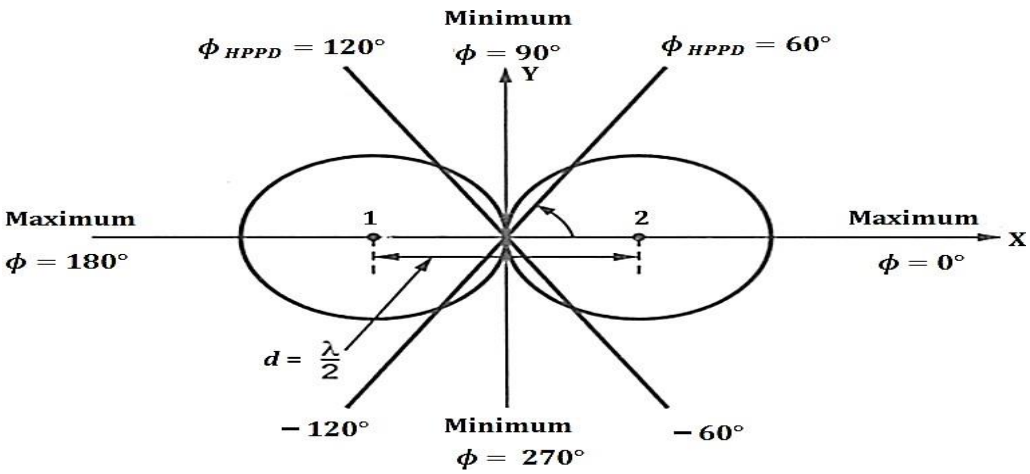


Fig. 3-6 Field pattern for two point source with $d = \lambda/2$ and fed with currents equal in magnitude and out of phase.

⇨ **<u>Two Point Sources with Currents Unequal in Magnitudeand with any Phase:</u>**

- Let us consider, two point sources are separated by distance d and supplied with currents which are different in magnitudes and with any phase difference say α, as shown in Fig. 3-7 (a).
- Assume that source 1 is taken as reference for phase. The amplitude of the fields due to source 1 and source 2 at the distant point P is E_1 and E_2 respectively, in which E_1 is greater than E_2, as shown in the vector diagram in Fig. 3-7 (b).

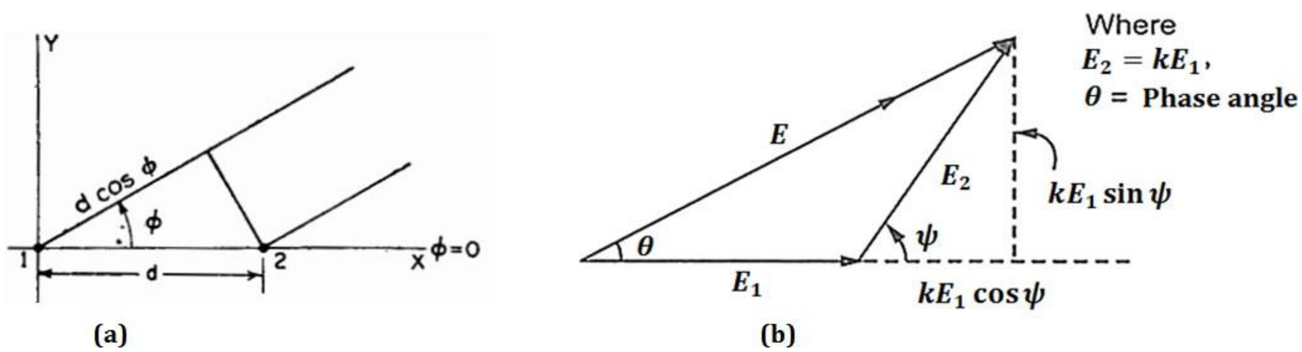


Fig. 3-7 (a) Two point sources with currents unequal in magnitude and with any phase (b) Vector diagram

- Now the total phase difference between the radiations by the two point sources at any far point P is given by,

$$\psi = kd\cos\phi + \alpha \qquad \text{-------- (3.22)}$$

where α is the phase angle with which current I_2 leads current I_1.

Then the resultant field at point P is given by,

$$E_T = E_1 e^{j0} + E_2 e^{j\psi} \qquad \text{-------- (3.23)}$$

(Source 1 is assumed to be reference hence phase angle is 0)

$$E_T = E_1 + E_2 e^{j\psi} = E_1\left(1 + \frac{E_2}{E_1} e^{j\psi}\right)$$

Let $\frac{E_2}{E_1} = k$ $\qquad E_T = E_1[1 + k(\cos\psi) + j\sin\psi]$ -------- (3.24)

Note that $E_1 > E_2$, the value of k is less than unity and varies from $0 \le k \le 1$.

- The magnitude and phase angle of the resultant field at point P is given by,

$$|E_T| = E_1\sqrt{(1 + k\cos\psi)^2 + j(k\sin\psi)^2} \qquad \text{-------- (3.25)}$$

$$\theta = \tan^{-1}\frac{k\sin\psi}{1 + k\cos\psi} \qquad \text{-------- (3.26)}$$

N- ELEMENT UNIFORM LINEAR ARRAY:

- An array of N elements is said to be linear array if all the individual elements are spaced equally along a line. An array is said to be uniform array if the elements in the array are fed with currents with equal magnitudes and with uniform progressive phase shift along the line.
- Consider uniform linear array of $\boldsymbol{N}$ isotropic point sources with all the individual elements spaced equally at distance $'d'$ from each other and all elements are fed with currents equal in magnitude and uniform progressive phase shift along line as shown in Fig. 3-8.

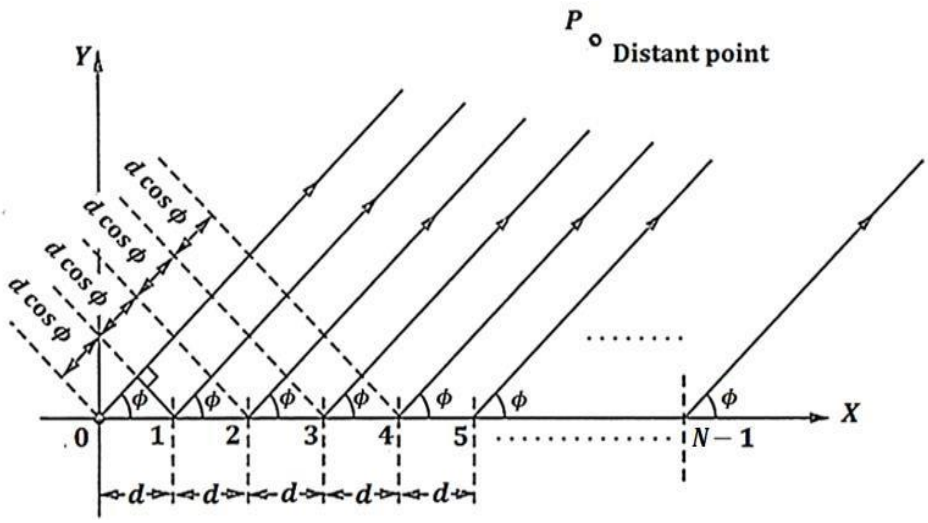


Fig. 3-8 Uniform linear array of *N* elements

- The total resultant field at the distant point P is obtained by adding the fields due to N individual sources vectorically. Hence,

$$E_T = E_0 e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \cdots\cdots\cdots + E_0 e^{(N-1)j\psi}$$

$$E_T = E_0(1 + e^{j\psi} + e^{2j\psi} + \cdots\cdots\cdots + e^{(N-1)j\psi}) \quad \text{-------- (3.27)}$$

- Note that $\psi = kd \cos\phi + \alpha$ indicates the total phase difference of the fields from adjacent sources calculated at point . Simlarly α is the progressive phase shift betweentwo adjacentpoint sources. The value of α may lie between 0°and 180°. If $\alpha = 0°$ we get N element uniform linear broadside array. If $\alpha = 180°$, we get N element uniformlinear end firearray.

  Multiply by $e^{j\psi}$ on both sides ;

$$E_T e^{j\psi} = E_0(e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \cdots\cdots\cdots + e^{Nj\psi}) \quad \text{-------- (3.28)}$$

- Subtract Eqn. (3.27) and (3.28) ;

$$E_T(1 - e^{j\psi}) = E_0(1 - e^{Nj\psi})$$

$$\frac{E_T}{E_0} = \frac{(1 - e^{Nj\psi})}{(1 - e^{j\psi}} = e^{j[(N-1)/2]\psi}\left[\frac{e^{j(N/2)\psi} - e^{-j(N/2)\psi}}{e^{j(1/2)\psi} - e^{-j(1/2)\psi}}\right] \quad \text{-------- (3.29)}$$

- If the reference point is the physical center of the array, then Eqn. (2.94)reduces to ;

$$\frac{E_T}{E_0} = AF = \left[\frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{1}{2}\psi)}\right]$$

  which is the ***antenna array factor.***

- If $\psi = 0$, the maximum value of E_T is determined using L'Hospital's rule ;

$$E_{T\,ma} = E_0\; N$$

- Thus the maximum value of E_T is N times the field from a single source. To normalize the field pattern , so that the maximum value of each is equal to unity. The normalized field pattern is given by ;

$$(E_T)_N = \frac{E_T}{E_{T\,max}} = \left[\frac{\sin(\frac{N}{2}\psi)}{N\;\sin(\frac{1}{2}\psi)}\right] \quad \text{-------- (3.30)}$$

- Eqn. (3.30) is the normalized array factor ;

$$(AF)_N = \left[\frac{\sin(\frac{N}{2}\psi)}{N\sin(\frac{1}{2}\psi)}\right] \quad \text{-------- (3.31)}$$

BROADSIDE ARRAY (BSA)

- An array is said to be broadside array, if maximum radiation occurs in direction perpendicular to array axis.
- In broadside array, individual elements are equally spaced along a line and each element is fed with current of equal magnitude and same phase.
- The total phase difference of the fields at point P from adjacent sources is given by,

$$\psi = kd\cos\phi + \alpha \qquad \text{-------- (3.32)}$$

- The normalized array factor for $'N'$ elements ;

$$(AF)_N = \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin\left(\frac{1}{2}\psi\right)}\right] \qquad \text{-------- (3.33)}$$

Major lobe

- In broadside array sources should be in phase i.e., $\alpha = 0°$ and $\psi = 0$ for maximum must be satisfied.

$$\psi = kd\cos\phi + \alpha = 0$$
$$kd\cos\phi + \alpha = 0 \qquad \because \alpha = 0$$
$$\cos\phi = 0 \quad ; \qquad \phi_m = 90° \text{ or } 270°$$

Nulls

- To find the nulls of the array Eqn. (3.33) is set to zero ;

$$k = \frac{2\pi}{\lambda}$$

$$\sin\left(\frac{N}{2}\psi\right) = 0 \quad ⇨ \quad \frac{N}{2}\psi\Big|_{\phi=\phi_n} = \pm n\pi$$

For BSA $\alpha = 0°$

$$\frac{N}{2}(kd\cos\phi_n + \alpha) = \pm n\pi \quad ⇨ \quad \phi_n = \cos^{-1}\left(\pm\frac{n\lambda}{Nd}\right) \qquad \text{-------- (3.34)}$$

where ; $n = 1, 2, 3 \ldots$

Maxima of minor lobes (secondary maxima)

- The maximum value of Eqn. (3.33) occur when ;

$$k = \frac{2\pi}{\lambda}$$

$$\sin\left(\frac{N}{2}\psi\right) = 1 \quad ⇨ \quad \frac{N}{2}\psi\Big|_{\phi=\phi_s} = \pm(2s+1)\frac{\pi}{2}$$

$$\frac{N}{2}(kd\cos\phi_s + \alpha) = \pm(2s+1)\frac{\pi}{2}$$

For BSA $\alpha = 0°$

$$\phi_s = \cos^{-1}\left\{\frac{1}{kd}\left[\pm\frac{(2s+1)\pi}{N} - \alpha\right]\right\}$$

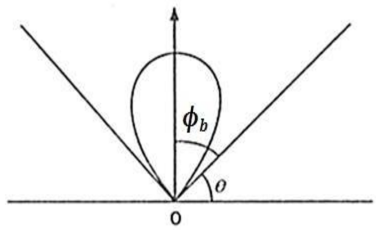
$$\phi_s = \cos^{-1}\left\{\frac{1}{kd}\left[\pm\frac{(2s+1)\pi}{N}\right]\right\} \qquad s = 1,2,3, \ldots$$

$$\phi_s = \cos^{-1}\left[\pm\frac{(2s+1)\lambda}{2Nd}\right] \qquad \text{-------- (3.35)}$$

Beamwidth of major lobe

- Beamwidth is defined as angle between first null and maximum of major lobe (or) Beamwidth is the angle equal to twice the angle between first null and the major lobe maximum.
$BWFN = 2 \times \phi_b = 2 \times (90 - \phi_n)$

$$90 - \phi_n = \phi_b \quad ⇨ \quad 90 - \phi_b = \phi_n \qquad \text{-------- (3.36)}$$



$$90 - \phi_b = \cos^{-1}\left(\pm \frac{n\lambda}{Nd}\right)$$

Take cosine on both sides ;

$$\cos(90 - \phi_b) = \cos\left(\cos^{-1}\left(\pm \frac{n\lambda}{Nd}\right)\right)$$

$$\sin \phi_b = \pm \frac{n\lambda}{Nd}$$

For first null $n = 1$

$$\sin \phi_b = + \frac{\lambda}{Nd}$$

Nd indicates the total length of the array L

$$BWFN = 2 \times \phi_b = + \frac{2\lambda}{Nd} \qquad \text{-------- (3.37)}$$

$$BWFN = \frac{2\lambda}{L} = \frac{2}{(L/\lambda)} \quad rad$$

$$BWFN = \frac{114.6°}{(L/\lambda)} \quad deg \qquad \text{-------- (3.38)}$$

Half power beamwidth (HPBW)

$$HPBW = \frac{BWFN}{2} = \frac{1}{(L/\lambda)} \quad rad$$

$$HPBW = \frac{57.3°}{(L/\lambda)} \quad deg \qquad \text{-------- (3.39)}$$

Directivity

- Directivity can be expressed in terms of the total length of the array ;

$$D_{max} = 2(L/\lambda) \qquad \text{-------- (3.40)}$$

Example: A broadside array of identical antennas consists of isotropic radiators separated by a distance $\boldsymbol{d = \lambda/2}$. Obtain positions of maxima and minima of the radiation pattern.

Solution:

Length of the array: $Nd = 4\left(\frac{\lambda}{2}\right) = \mathbf{2\lambda}$

Major lobe: $\phi_m = 90°$ or $270°$

Maxima of minor lobes (Secondary maxima):

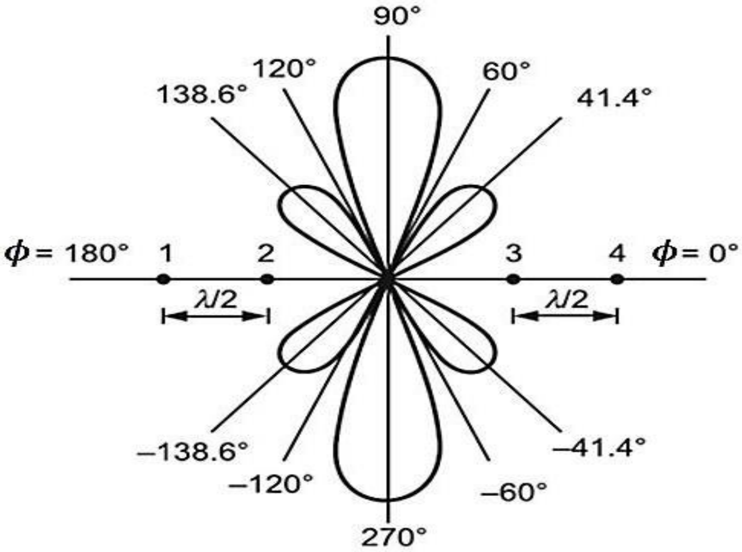
$$\phi_s = \cos^{-1}\left[\pm \frac{(2s+1)\lambda}{2Nd}\right] \qquad s = 1,2,3, \ldots \qquad s = 1\,;\ \pm 41.4°, \pm 138.6°$$

∴ These are the 4 minor lobe maxima of the array of 4 isotropic radiators fed in phase and spaced $\boldsymbol{\lambda/2}$ apart. No other maxima exist for $s \geq 2$, because for $s = 2$, $\cos^{-1}\left(\pm \frac{5}{4} \gg 1\right)$ whereas cosine value is always << 1.

Nulls:

$$\phi_n = \cos^{-1}\left(\pm \frac{n\lambda}{Nd}\right) \qquad n = 1,2,3,\ldots \qquad n = 1\,;\ \pm 60°,\ \pm 120°$$
$$n = 2\,;\ \pm 0°,\ \pm 180°$$

∴ 0°, 60°, 120°, 180°, −60°, −120° are six minor lobe minima of the array of 4 isotropic radiators spaced $\boldsymbol{\lambda/2}$ apart. No other minima (nulls) exist for which cosine functions becomes more than one.



END-FIRE ARRAY (EFA)

- An array is said to be end-fire array, if the direction of maximum radiation coincides with the array axis.
- In end-fire array, individual elements are equally spaced along a line and each element is fed with current of equal magnitude and opposite phase.
- The total phase difference of the fields at point P from adjacent sources is given by,

$$\psi = kd\cos\phi + \alpha \qquad \text{-------- (3.41)}$$

- The normalized array factor for $'N'$ elements ;

$$(AF)_N = \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin\left(\frac{1}{2}\psi\right)}\right] \qquad \text{-------- (3.42)}$$

Major lobe

- In end-fire array $\psi = 0$ and $\phi = 0°$ or $180°$

$$\psi = kd\cos\phi + \alpha = 0$$

$\psi = 0$ and $\phi = 0°$ ⇨ $\alpha = -kd$ -------- (3.43)

$\psi = 0$ and $\phi = 180°$ ⇨ $\alpha = kd$ -------- (3.44)

$$\phi_m = 0° \text{ or } 180°$$

Nulls

- To find the nulls of the array Eqn. (3.42) is set to zero ;

$$k = \frac{2\pi}{\lambda}$$

$$\sin\left(\frac{N}{2}\psi\right) = 0 \quad \Rightarrow \quad \frac{N}{2}\psi\Big|_{\phi=\phi_n} = \pm n\pi$$

For EFA $\alpha = -kd$

$$\frac{N}{2}(kd\cos\phi_n + \alpha) = \pm n\pi$$

$$\frac{N}{2}(kd\cos\phi_n - kd) = \pm n\pi \qquad \text{where ; } n = 1, 2, 3 \ldots$$

$$\frac{Nd}{\lambda}(\cos\phi_n - 1) = \pm n \qquad \text{-------- (3.45)}$$

$$2\sin^2\frac{\phi_n}{2} = \pm\frac{n\lambda}{Nd}$$

$$\phi_n = 2\sin^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right) \qquad \text{-------- (3.46)}$$

Further simplication : From Eqn. (3.45) , note that the value of $(\cos\phi_n - 1)$ is always less than 1, Hence it is negative. So consider negative values of R.H.S ;

From Eqn. (3.45):

$$\frac{Nd}{\lambda}(\cos\phi_n - 1) = -n$$

$$\phi_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right) \qquad \text{-------- (3.47)}$$

Maxima of minor lobes (secondary maxima)

- The maximum value of Eqn. (3.42) occur when ;

$$k = \frac{2\pi}{\lambda}$$

$$\sin\left(\frac{N}{2}\psi\right) = 1 \quad \Rightarrow \quad \frac{N}{2}\psi\Big|_{\phi=\phi_s} = \pm(2s+1)\frac{\pi}{2}$$

$$\frac{N}{2}(kd\cos\phi_s + \alpha) = \pm(2s+1)\frac{\pi}{2}$$

For EFA $\alpha = -kd$

$$(\cos\phi_s - 1) = \pm(2s+1)\frac{\lambda}{2Nd} \quad s = 1,2,3,\ldots \qquad \text{-------- (3.48)}$$

$$\phi_s = \cos^{-1}\left[\pm(2s+1)\frac{\lambda}{2Nd} + 1\right] \qquad \text{-------- (3.49)}$$

Further simplication : From Eqn. (3.48) , note that the value of $(\cos\phi_s - 1)$ is always less than 1, Hence it is negative. So consider negative values of R.H.S ;

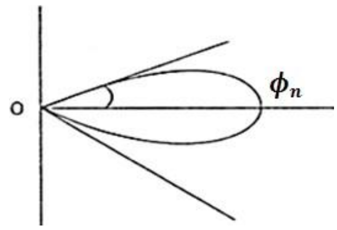
From Eqn. (3.48):

$$(\cos\phi_s - 1) = -(2s+1)\frac{\lambda}{2Nd}$$

$$\phi_s = \cos^{-1}\left[1 - \frac{(2s+1)\lambda}{2Nd}\right] \qquad \text{-------- (3.50)}$$

Beamwidth of major lobe

- Beamwidth is defined as angle between first null and maximum of major lobe (or) Beamwidth is the angle equal to twice the angle between first null and the major lobe maximum.
$BWFN = 2 \times \phi_n$



$$\phi_n = 2\sin^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right)$$

$$\sin\frac{\phi_n}{2} = \pm\sqrt{\frac{n\lambda}{2Nd}}$$

For small angles ; $\sin\phi_n \approx \phi_n$

$$\frac{\phi_n}{2} \approx \pm\sqrt{\frac{n\lambda}{2Nd}}$$

$$\phi_n = \pm\sqrt{\frac{2n\lambda}{Nd}} \qquad \text{-------- (3.51)}$$

Nd indicates the total length of the array L

$$\phi_n = \pm\sqrt{\frac{2n\lambda}{Nd}}$$

For first null $n = 1$

$$\phi_n = \pm\sqrt{\frac{2\lambda}{L}}$$

$$BWFN = 2 \times \phi_n = \pm 2\sqrt{\frac{2\lambda}{L}} \qquad \text{-------- (3.52)}$$

$$BWFN = \pm 2\sqrt{\frac{2}{(L/\lambda)}}\ rad$$

$$BWFN = 114.6^\circ\sqrt{\frac{2}{(L/\lambda)}}\ deg \qquad \text{-------- (3.53)}$$

Half power beamwidth (HPBW)

$$HPBW = \frac{BWFN}{2} = \pm\sqrt{\frac{2}{(L/\lambda)}}$$

$$HPBW = 57.3^\circ\sqrt{\frac{2}{(L/\lambda)}}\ deg \qquad \text{-------- (3.54)}$$

Directivity

- Directivity can be expressed in terms of the total length of the array ;

$$D_{max} = 4(L/\lambda) \qquad \text{-------- (3.55)}$$

Example: A end-fire array of identical antennas consists of isotropic radiators separated by a distance $\boldsymbol{d = \lambda/2}$. Obtain positions of maxima and minima of the radiation pattern.

Solution:

Length of the array: $Nd = \frac{\lambda}{2}$

Major lobe: $\phi_m = 0°$ or $180°$

Maxima of minor lobes (Secondary maxima):

$\phi_s = \cos^{-1}[1 - (2s+1)\frac{\lambda}{2Nd}]$ $s = 1,2,3, \ldots$

$s = 1$; $\pm 75.5°$

$s = 2$; $\pm 104.5°$

$s = 3$; $\pm 138.6°$

∴ These are the 6 minor lobe maxima of the array of 4 isotropic radiators fed spaced $\boldsymbol{\lambda/2}$ apart. No other maxima exist for $s \geq 4$, because for $s = 4$, $\cos^{-1}(\pm \frac{5}{4}) \gg 1$, whereas cosine value is always << 1.

Nulls:

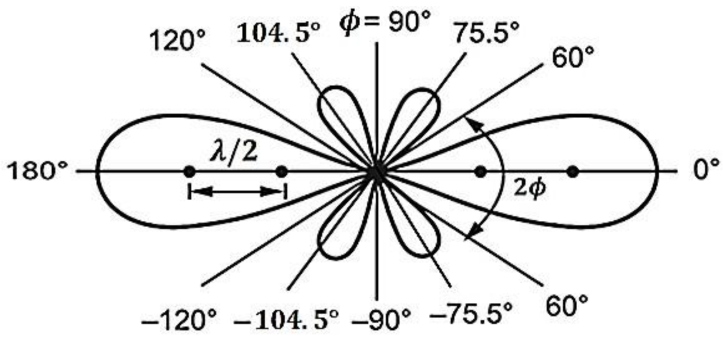
$\phi_n = \cos^{-1}(1 - \frac{n\lambda}{Nd})$ $n = 1,2,3, \ldots$

$n = 1$; $\pm 60°$

$n = 2$; $\pm 90°$

$n = 3$; $\pm 120°$

∴ $\pm 60°, \pm 90°, \pm 120°$ are six minor lobe minima of the array of 4 isotropic radiators spaced $\boldsymbol{\lambda/2}$ apart. No other minima (nulls) exist for which cosine functions becomes more than one.



HANSEN WOODYARD END-FIRE ARRAY

- In end-fire array, the maximum radiation can be obtained along the axis of the array , if the progressive phase shift α between the elements is given by ;

$= \pm\, kd$; $\alpha = -kd$ for maximum $\phi = 0°$ direction
$\alpha = kd$ for maximum $\phi = 180°$direction

- It is found that the field produced in the direction $\phi = 0°$is maximum, but the directivity is not maximum. In many applications it is necessary to have the maximum possible directivity of the linear array.

- In 1938, Hansen and Woodyard proposed certain conditions for the end-fire case which are helpful in enhancing the directivity without altering other characteistics of the end-fire array. These conditions are known as Hansen –Woodyard conditions for end-fire radiation.
- According to Hansen –Woodyard conditions, the phase shift between closely spaced radiators of a very long array should be ;

$$\alpha = -(kd + 2.94/n) \approx -(kd + \pi/n) \text{ for maximum } \phi = 0° \text{ direction}$$

$$\alpha = +(kd + 2.94/n) \approx +(kd + \pi/n) \text{ for maximum } \phi = 180° \text{ direction}$$

Directivity

- Directivity can be expressed in terms of the total length of the array ;

$$D_{max} = 1.789[4(L/\lambda)] \quad \text{-------- (3.56)}$$

Parameter :	Broadside array	End-fire array	Hansen –Woodyard End-fire array
Directivity	$D_{max} = 2(L/\lambda)$	$D_{max} = 4(L/\lambda)$	$D_{max} = 1.789[4(L/\lambda)]$
	Where ; $L = Nd$		

PRINCIPLE OF PATTERN MULTIPLICATION

- *The field pattern of an array of non-isotropic but similar sources is the product of the pattern of the individual sources and the pattern of isotropic point sources having the same locations, relative amplitudes, and phase as the non-isotropic point sources.* This is referred to as pattern multiplication for arrays of identical elements.

$$\text{Total field } (E) = \underbrace{\{E_i(\theta,\phi) \times E_a(\theta,\phi)\}}_{\text{Multiplication of Field pattern}} \times \underbrace{\{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)\}}_{\text{Addition of Phase Pattern}} \quad \text{-------- (3.57)}$$

where ; $E_i(\theta,\phi)$ = Field pattern of individual source

$E_a(\theta,\phi)$ = Field pattern of array of isotropic source

$E_{pi}(\theta,\phi)$ = Phase pattern of individual source

$E_{pa}(\theta,\phi)$ = Phase pattern of array of isotropic source

⇨ ***RADIATION PATTERN OF 4-ISOTROPIC ELEMENTS FED IN PHASE & SPACED $\lambda/2$ APART***

- Consider a 4-element array of antennas as shown in Fig. 3-8, in which the spacing between the elements is $\lambda/2$ and the currents are in-phase ($\alpha = 0$). The pattern can be obtained directly by adding the four electric fields due to the 4 antennas. However the same radiation pattern can be obtained by pattern multiplication in the following manner.

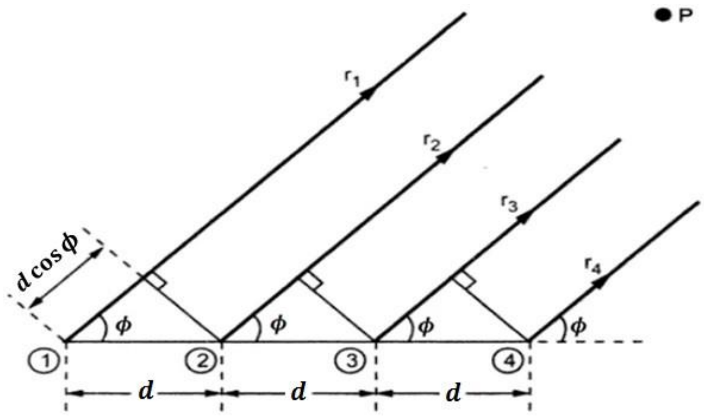
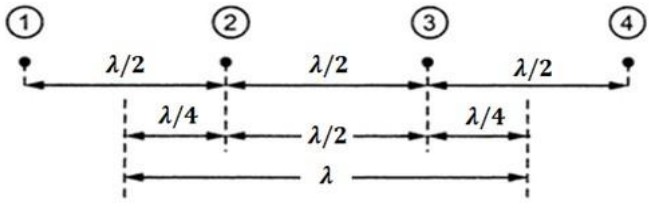
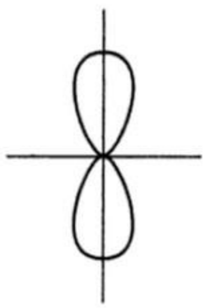


Fig. 3-8 Linear array of 4 isotropic elements spaced $\lambda/2$ apart , fed in-phase

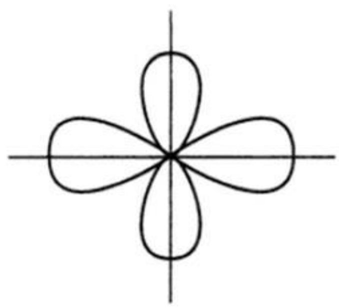
- Two isotropic point sources spaced $\lambda/2$ apart fed in-phase provides a bidirectional pattern as in Fig. 3-9 (b). Now the elements 1 and 2 are considered as one unit and this new unit is considered to be placed between the midway of elements 1, 2 and similarly the elements 3,4 as shown in Fig. 3-9 (a).



(a) Antenna ① and ② and ③ and ④ replaced by single antenna separately



(b) Radiation pattern of 2 antennas spaced at distance $\lambda/2$ and fed with equal currents in phase



(c) Radiation pattern of 2 antennas spaced at distance λ and fed with equal currents in phase

Fig. 3-9

- 4 elements spaced $\lambda/2$ have been replaced by 2 units spaced λ and therefore the problem of determining radiation of 4 elements has been reduced to find out the radiation pattern of 2 antennas spaced λ apart as in Fig. 3-9 (a).

$$\left\{\begin{matrix} Resultant\ radiation \\ pattern\ of\ 4\ elements \end{matrix}\right\} = \left\{\begin{matrix} Resultant\ radiation \\ pattern\ of\ individual\ elements \end{matrix}\right\} \times \left\{\begin{matrix} rray\ of\ 2\ units \\ spaced\ \lambda \end{matrix}\right\}$$

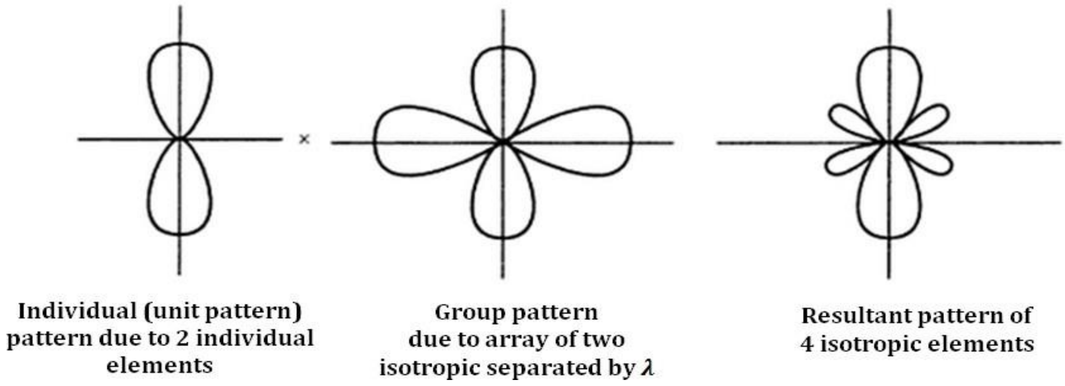


Fig. 3-10 Resultant radiation pattern of 4 isotropic elements by pattern multiplication

- Here the width of the principal lobe is the same as the width of the corresponding lobe of the group pattern. The number of secondary lobes can be determined from the nulls in the resultant pattern, which is sum of the nulls in the unit and group patterns.

CONCEPT OF PHASED ARRAYS, ADAPTIVE ARRAY

- In case of the broadside array and the end fire array, the maximum radiation can be obtained by adjusting the phase excitation beween elements in the direction normal and along the axis of array respectively.
- That means in other words elements of antenna array can be phased in particular way. So we can obtain an array which gives maximum raidation in any direction by controlling phase excitation in each element. Such an array is commonly called phased array.
- The array in which the phase and the amplitude of most of the elements is variable, provided that the direction of maximum radiation (beam direction) and pattern shape along with the side lobes is controlled, is called as phased array.
- Suppose the array gives maximum radiation in direction $\phi = \phi_0$ where $0 \leq \phi_0 \leq 180°$, then the phase shift that must be controlled can be obtained as follows.

$$\psi = kd \cos \phi + \alpha|_{\phi=\phi_0} = 0 \qquad \text{-------- (3.58)}$$

- Thus from Eqn. (3.58), it is clear that the maximum radiation can be achieved in any direction if the progressive phase difference between the elements is controlled. The electronic phased array operates on the same principle.
- Consider a three element array as shown in the Fig. 3-11. The element of array is considered as $\lambda/2$ dipole. All the cables used are of same length. All the three cables are brought together at common feed point. Here mechanical switches are used. Such switch is installed one at each antenna and one at a common feed point.
- All the switches are ganged together. Thus by operating switch, the beam can be shifted to any phase shift.
- To make operation reliable and simple, the ganged mechanical switch is replaced by PIN dipole which acts as electronic switch. But for precision in results, the number of cables should be minimised.

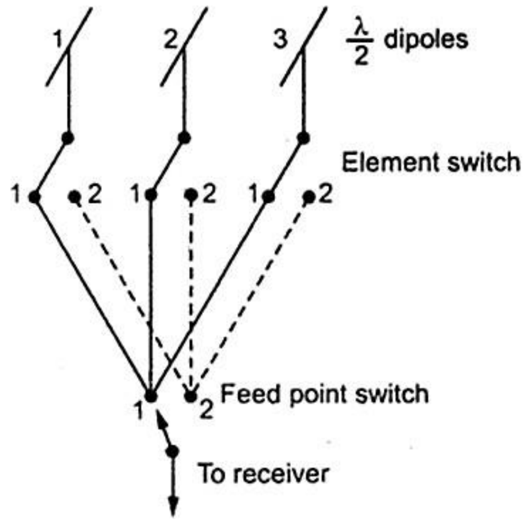


Fig. 3-11 Phased array with mechanical switches at elements and feed point

- To make operation reliable and simple, the ganged mechanical switch is replaced by PIN dipole which acts as electronic switch. But for precision in results, the number of cables should be minimised.
- In many applications phase shifter is used instead of controlling phase by switching cables. It can be achieved by using ferrite device. The conducting wires are wrapped around the phase shifter. The current flowing through these wires controls the magnetic field within ferrite and then the magnetic field in the ferrite controls the phase shift.
- The phased array for specialized functional utility are recognized by different names such as frequency scanning array, retroarray and adoptive array.
- The array in which the phase change is controlled by varying the frequency is called frequency scanning array. This is found to be the simplest phased array as at each element separate phase control is not necessary. A simple transmission line fed frequency scanning array as shown in the Fig. 3-12.

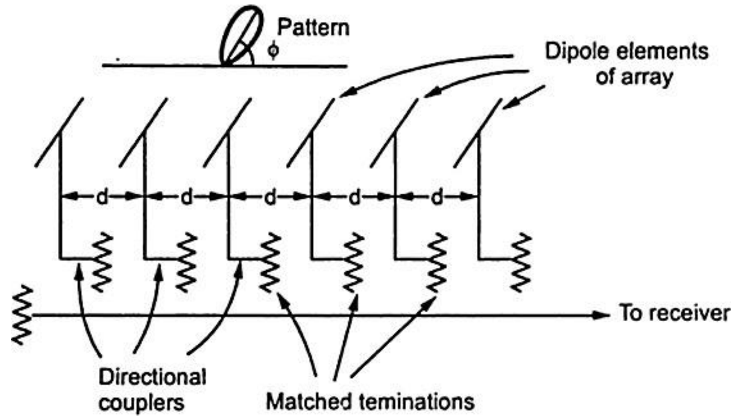


Fig. 3-12 Frequency scanning line fed phased array.

- Each element of the scanning array is fed by a transmission line via directional coupler. Note that the directional couplers are fixed in position, while the beam scanning is done with a frequency change. To avoid reflections and to obtain pure form of the travelling wave, the transmission line is properly terminated of the load.

- The main advantage of the frequency scaning array is that there are no moving parts and no switches and phase shifters are required.
- The array which automatically reflects an incoming signal back to the source is called retroarray. It acts as a retroreflector similar to the passive square corner reflector. That means the wave incident on the array is received and transmitted back in the same direction.
- In other words, each element of the retroarray reradiates signal which is actually the conjugate of the received one. Simplest form of the retroarray is the Van Atta array as X shown in the Fig. 3-13 in which 8 identical dipole elements are used, with pairs formed between elements l and 8, 2 and 7, 3 and 6, 4 and 5 using cables of equal length. If the wave arrives at angle say , then it gets transmitted in the same direction.

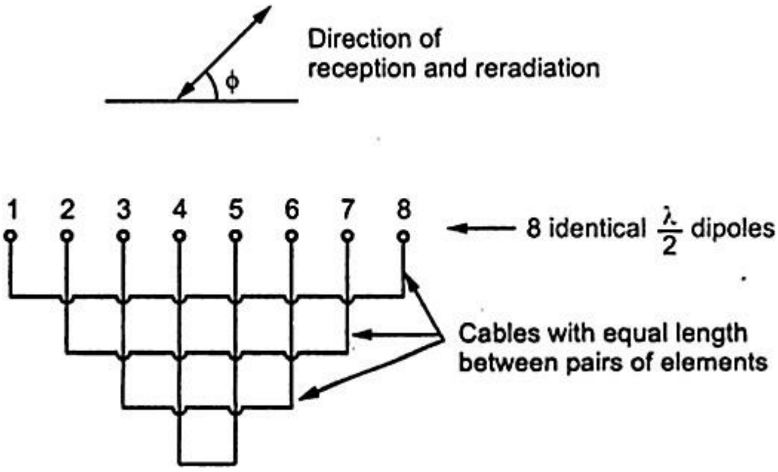


Fig. 3.15 Van Atta retorarray

- An array which automatically turn the maximum beam in the desired direction while turn the null in the undesired direction is called adoptive array. The adpotive array adjust itself in the desired direction with awareness of its enviomment.
- In modem adoptive arrays, the output of each element in the array is sampled, digitized and then processed using computers. Such arrays are commonly called smart antennas.

BASIC PRINCIPLE OF ANTENNA SYNTHESIS - (BINOMIAL ARRAY)

- In case of uniform linear array, to increase the directivity, the array length has to be increased. But when the array length increases, the secondary or side lobes appear in the pattern. In some of the special applications, it is desired to have single main lobe with no minor lobes.
- That means the minor lobes should be eliminated completely or reduced to minimum level as compared to main lobe. To achieve such pattern, the array is arranged in such away that the broad side array radiate more strongly at the centre than that from edges.
- To reduce the side lobe level, John Stone proposed that sources have amplitudes proportional to the coefficients of a binomial series of the form ;

$$(1+x)^{m-1} = 1 + (m-1)x + \frac{(m-1)(m-2)}{2!}x^2 + \frac{(m-1)(m-2)(m-3)}{3!}x^3 + \ldots \quad \text{-------- (3.59)}$$

where m is the number of radiating sources in the array.

Binomial array is an array whose elements are excited according to the current levels determined by the binomial coefficient.

- The positive cooefficients of the series expansion for different values of m are ;

$m=1$										1										
$m=2$									1		1									
$m=3$								1		2		1								
$m=4$							1		3		3		1							
$m=5$						1		4		6		4		1						
$m=6$					1		5		10		10		5		1					
$m=7$				1		6		15		20		15		6		1				
$m=8$			1		7		21		35		35		21		7		1			
$m=9$		1		8		28		56		70		56		28		8		1		
$m=10$	1		9		36		84		126		126		84		36		9		1	

-------- (3.60)

- The above represents Pascal's triangle. If the values of m are used to represent the number of elements of the array, then the coefficients of the expansion represent the relative amplitudes of the elements. Since the coefficients are determined from a binomial series expansion, the array is known as binomial array.

Nonuniform amplitude arrays of even and odd number of elements:

- Assuming that the amplitude excitation is symmetrical about the origin, the array factor for a nonuniform amplitude array can be written as ;

$$(AF)_{2M}(\text{even}) = \sum_{n=1}^{M} a_n \cos[(2n-1)u]$$

$$(AF)_{2M+1}(\text{odd}) = \sum_{n=1}^{M+1} a_n \cos[2(n-1)u] \quad \text{-------- (3.61)}$$

where

$$u = \frac{\pi d}{\lambda}\cos\phi$$

where a_n's are the excitation coefficients of the array elements.

- From Eqn's (3.60) and (3.61) , the amplitude coefficients for the following arrays are:

1. Two elements $(2M = 2)$

   $a_1 = 1$

2. Three elements $(2M + 1 = 3)$

   $2a_1 = 2 \Rightarrow a_1 = 1$

   $a_2 = 1$

3. Four elements $(2M = 4)$

   $a_1 = 3$

   $a_2 = 1$

4. Five elements $(2M + 1 = 5)$

   $2a_1 = 6 \Rightarrow a_1 = 3$

   $a_2 = 4$

   $a_3 = 1$

- Binomial array's do not exhibit any minor lobes provided the spacing between the elements is equal or less than one-half of a wavelength.
- The design using a $\lambda/2$ spacing leads to a pattern with no minor lobes, the half-power beamwidth and maximum directivity for $d = \lambda/2$ spacing in terms of the numbers of elements or the length of the array are given by ;

$$HPBW\ (d = \lambda/2) \approx \frac{0.75}{\sqrt{L/\lambda}} \qquad \text{-------- (3.62)}$$

$$D_{max} = 1.77\sqrt{1 + 2L/\lambda} \qquad \text{-------- (3.63)}$$

- The advantages of binomial array is that there are no side lobes in the resultant pattern. The disadvantages are i. Beam width of the main lobe is large which is undesirable ii. Directivity is small and high excitation levels are required for the center elements of large arrays.

Yagi Arrays:

Yagi Uda Antenna:

Yagi-Uda arrays or Yagi-Uda antennas are high gain antennas. The antenna was first invented by a Japanese Prof. S. Uda in early 1940's and described in English by Prof. H. Yagi. Hence the antenna name Yagi-Uda antenna was given after Prof. S. Uda and Prof. H. Yagi. A basic Yagi-Uda antenna consists a driven element, one reflector and one or more directors. Basically it is an array of one driven element and one of more parasitic elements. The driven element is a folded dipole made of a metallic rod which is excited.

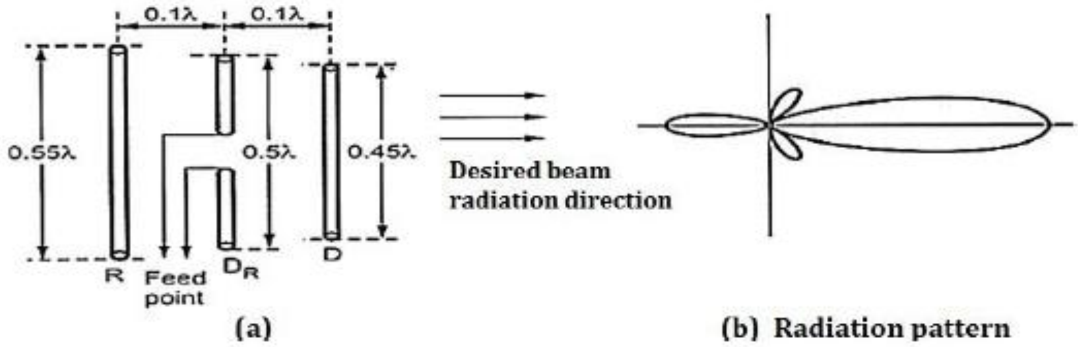
A Yagi-Uda antenna uses both the reflector (R) and the director (D) elements in same antenna.The element at the back side of the driven element is the reflector. It is of the larger length compared with remaining elements. The element in front of the driven element is the director which is of lowest length. Directors and reflector are called parasitic elements. All the elements are placed parallel and close to each other as shown in Fig. 1. The length of the folded dipole is about $\lambda/2$ and it is at resonance. Length of the director is less than $\lambda/2$ and length of the reflector is greater than $\lambda/2$.

The parasitic element receive excitation through the induced e.m.f. as current flows in the driven element.The phase and amplitude of the currents through the parasitic elements mainly depends on the length of the elements and spacing between the elements. To vary reactance of any element, the dimensions of the elements are readjusted. Generally the spacing between the driven and the parasitic elements is kept nearly 0.1 λ to 0.15 λ.

$$\text{Reflector length} = \frac{152}{f_{MHz}} \ \text{meter}$$

$$\text{Driver element length} = \frac{143}{f_{MHz}} \ \text{meter}$$

$$\text{Director length} = \frac{137}{f_{MHz}} \ \text{meter}$$



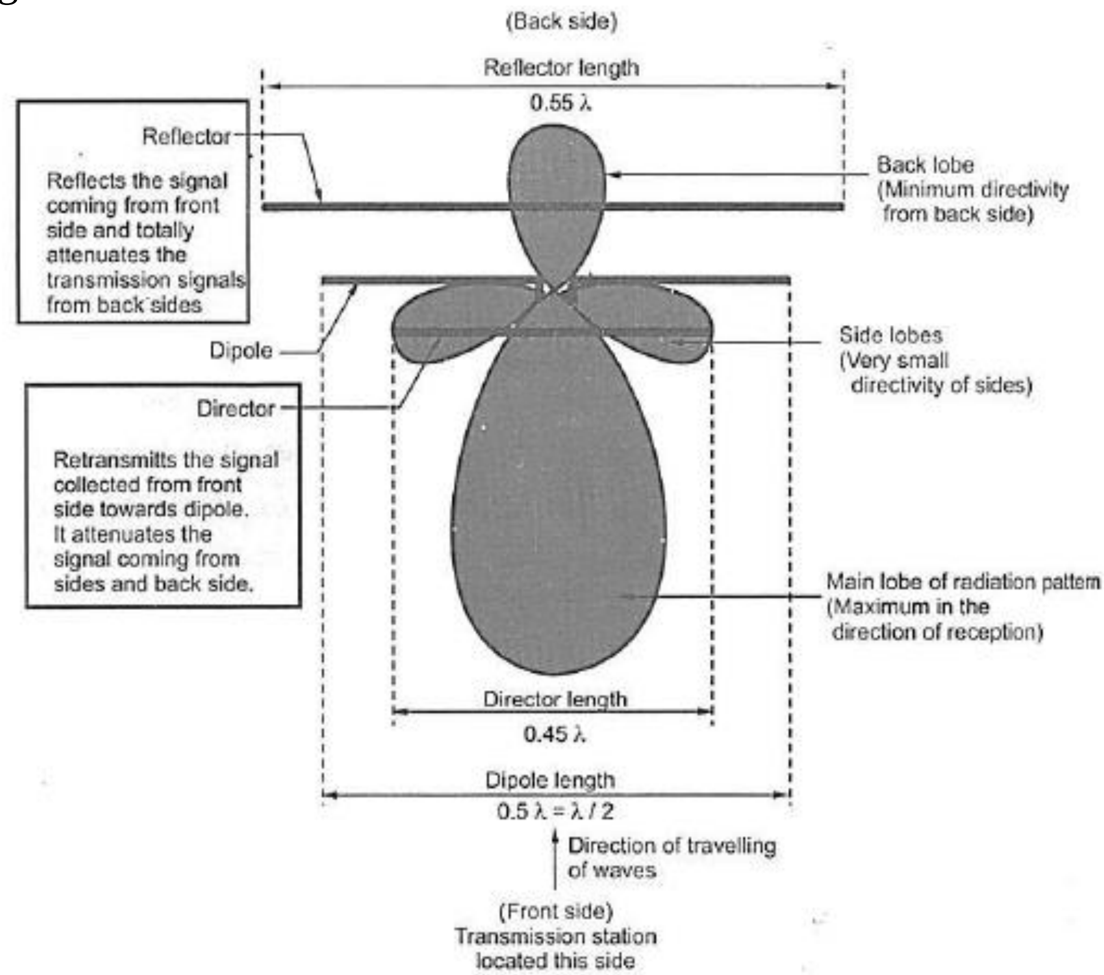
Working of Yagi Antenna:

The parasitic element is used either to direct or to reflect, the radiated energy forming compact directional antenna. If the parasitic element is greater than length $\lambda/2$, (i.e. reflector) then it is inductive in nature. Hence the phase of the current in such element lags the induced voltage.If the parasitic element is less than resonant length $\lambda/2$ (i.e. director), then it is capacitive in nature.

Hence the current in director leads the induced voltage.The directors adds the fields of the driven element in the direction away from the driven element. If more than one directors are used, then each director will excite the next.To increase the gain of the Yagi-Uda antenna, the number of directors is increased in the beam direction.

To get good excitation, the elements are closely spaced.The driven element radiates from front to rear (i.e., from reflector to director). Part of this radiation induces currents in the parasitic elements which reradiate almost all radiations. With the proper lengths of the parasitic elements and the spacing between the elements, the backward radiation is cancelled and the radiated energy is added in front.

Yagi Antenna Radiation Pattern:



Applications of Yagi Antenna:

- Yagi-Uda array is the most popular antenna for the reception of terrestrial television signals in the VHF band (30 MHz-300 MHz).
- The array for this application is constructed using aluminium pipes.
- The driven element is usually a folded dipole, which gives four times the impedance of a standard dipole.
- Thus, a two-wire balanced transmission line having a characteristic impedance of 300 Ω can be directlyconnected to the input terminals of the Yagi-Uda array.
- Yagi-Uda arrays have been used in the HF, VHF, UHF, and microwave frequency bands.

Post – MCQ:

1. The antenna array is defined as the system of similar antennas directed to get required ------------------ in the desired direction.
   (a) High gain
   (b) High directivity
   (c) High bandwidth
   (d) All of the above

Answer - d

2. Find the odd element out –
   (a) Broadside array
   (b) End fire array
   (c) Co-linear array
   (d) Yagi array

Answer - d

3. The Broadside array is defined as an array having maximum radiation-------------------- the axis array
   a) Perpendicular to
   b) Along
   c) Parallel
   d) None of the above

Answer - a

4. The collinear array is also called as –
   (a) Broadcast array
   (b) Broad fire array
   (c) Omni directional array
   (d) All of the above

Answer - a

5. The End fire array is defined as an array having maximum radiation-------------------- the axis array

a) Perpendicular to
b) Along
c) Parallel
d) None of the above

Answer - b

6. An array is said to be uniform array if the array elements are fed with –
   (a) Equal amplitudes and any phase shift
   (b) Equal amplitudes and uniform progressive phase shift
   (c) Unequal amplitudes and any phase shift
   (d) None of the above

Answer - b

7. In a broadside array, the direction of maximum radiations indicated by –
   (a) 0 and 180 degrees
   (b) 90 and 270 degrees
   (c) 90 and 180 degrees
   (d) None of the above

Answer -b

8. In a End fire array, the direction of minimum radiations indicated by –
   (a) 0 and 180 degrees
   (b) 90 and 270 degrees
   (c) 90 and 180 degrees
   (d) None of the above

Answer - b

9. The relation between directivity and the array factor length is given by –
   (a) D = 2 (L/ƛ)
   (b) D = 3(L/ƛ)
   (c) D = 4(l/ƛ)
   (d) None of the above

Answer - c

10. In a phase array, the maximum radiation in any direction can be obtained by controlling ------------------ excitation in each element.
   (a) Angle
   (b) Phase
   (c) Amplitude
   (d) None of the above

Answer - b

11. The Adaptive array is an array which turns the --------------beam in the desired direction and ----------in the undesired direction.
   (a) Minimum and maximum
   (b) Maximum and zero
   (c) Maximum and minimum
   (d) None of the above

Answer - b

12. The frequency scanning array is an array in which phase change can be controlled by Varying the –

    (a) Phase
    (b) Frequency
    (c) amplitude
    (d) Any of the above

Answer -b

13. The array in which the incoming signal received and sent back in the same direction is called as-
    (a) End fire array
    (b) Frequency scanning array
    (c) Van Atta array
    (d) None of the above

Answer - c

14. Binomial array is an array whose elements are excited according to the current levels determined by the –
    (a) binomial coefficients
    (b) binary coefficients
    (c) integer coefficients
    (d) All of the above

Answer - a

15. The important characteristic of a binomial array is –
    (a) Small beam width
    (b) High directivity
    (c) No side lobes
    (d) All of the above

Answer - c

Conclusion:

At the end of the topic, students will be able –

1) To understand the basic concept of Antenna Arrays and their radiation characteristics.

2) To understand the principle of types of Antenna arrays

3) To get exposure to different types like Broadside array and End fire arrays etc

4) To know the applications of Antenna arrays – example Yagi uda array

References:

1. John D Kraus," Antennas for all Applications", 4^{th} Edition, McGraw Hill, 2010.
2. R.E. Collin," Antennas and Radio wave Propagation", McGraw Hill 1985.
3. Constantine.A. Balanis "Antenna Theory Analysis and Design", Wiley Student Edition, 4^{th} Edition 2016.
4. Rajeswari Chatterjee, "Antenna Theory and Practice" Revised Second Edition New Age International Publishers, 2006.

Assignments:

1. Explain the working principle of Broadside antenna array and derive an expression for the resultant electric field.
2. Explain the working principle of End fire antenna array and derive an expression for the resultant electric field
3. Derive an expression for electric field intensity of an array of N - isotropic sources of (i) Equal amplitude and same phase (ii) Equal amplitude and opposite phase.
4. Write a short note on antenna arrays.
5. Differentiate between BSA and EFA.

Subject Name: **Antennas & Propagation**

Topic Name: **Aperture and Slot Antennas**

(Unit – 3)

Syllabus / Aperture and Slot Antennas

1. Aperture Antennas - Horn antenna, Reflector antenna, Aperture blockage
2. Feeding structures, Slot antennas, Micro strip antennas, Radiation mechanism – Application
3. Numerical tool for antenna analysis

Aim and Objective:

- To give insight of basic Knowledge of Aperture Antennas
- To give thorough understanding of the radiation characteristics of Aperture antennas Horn antenna, Reflector antenna, Slot antenna, Micro strip antenna.
- To impart knowledge about different feeding structure Aperture antennas and applications.

Pre – Test MCQ:

1. The array factor is defined as factor by which array increases the field strength over that of –
   (a) Multiple elements radiating the same power
   (b) Single element radiating the same power
   (c) Half of elements radiating the same power
   (d) None of the above

Ans: b

2. An antenna array is said to be linear if all the array elements are –
   (a) Equally spaced in a straight line
   (b) Perpendicular to each other in a straight line
   (c) Unequally spaced in a straight line
   (d) None of the above

Ans: a

3. The principle of pattern multiplication states that multiplication of –
   (a) individual source patterns and isotropic source patterns
   (b) isotropic source patterns and individual source patterns
   (c) individual source patterns and identical source patterns
   (d) None of the above

Ans: a

4. The pattern multiplication is a method to sketch the radiation pattern of an array –
   (a) By calculating total electric field strength of the array
   (b) By inspection
   (c) By combining (a) and (b)
   (d) None of the above

Ans:b

5. The uniform amplitude distribution is a process by which sources are –
   (a) equal in amplitude and in phase
   (b) equal in amplitude and out of phase
   (c) unequal in amplitude and any phase
   (d) None of the above

Ans:a

6. The gain of an antenna array is the ability of the antenna to –
   (a) Concentrate the radiated power in a given direction
   (b) Absorb the incident power in a given direction
   (c) Both (a) and (b)
   (d) None of the above

Ans:c

7. The array in which the antenna elements are arranged end to end along a straight line in a co-axial manner is called as-
   (a) Broadside array
   (b) End fire array
   (c) Co-linear array
   (d) None of the above

Ans: c

8. In a parasitic array, the parasitic element gets the power from the driven element by means of –
   (a) Electric coupling
   (b) Magnetic coupling
   (c) Electro-magnetic coupling
   (d) None of the above

Ans: c

9. The tuning of an array is a process by which the relative phases of the currents are changed by adjusting the –
   (a) spacing between array elements
   (b) length of the elements
   (c) both (a) and (b)
   (d) None of the above

Ans: a

10. In an antenna array, the total electric field strength can be obtained by –
   (a) Adding the individual fields mathematically
   (b) Adding the individual fields vectorically
   (c) Adding the individual fields logically
   (d) None of the above

Ans: b

Pre-requisite

- Basic knowledge of Electromagnetic Fields and Wave guides.
- Basic Knowledge of Antenna Arrays and Antenna parameters

UNIT III: APERTURE AND SLOT ANTENNAS

Radiation from rectangular apertures, Uniform and Tapered aperture, Horn antenna, Reflector antenna , Aperture blockage , Feeding structures , Slot antennas, Microstrip antennas – Radiation mechanism – Application , Numerical tool for antenna analysis

APERTURE ANTENNAS:

- Aperture antennas are most common at microwave frequencies. In general aperture means opening. In concern with antenna, an aperture means opening in a closed surface.
- There are many different geometrical configurations of an aperture antenna with some of the most popular shown in Fig. 2-1. They may take the form of a waveguide or a horn whose aperture may be square, rectangular, circular, elliptical, or any other configuration.

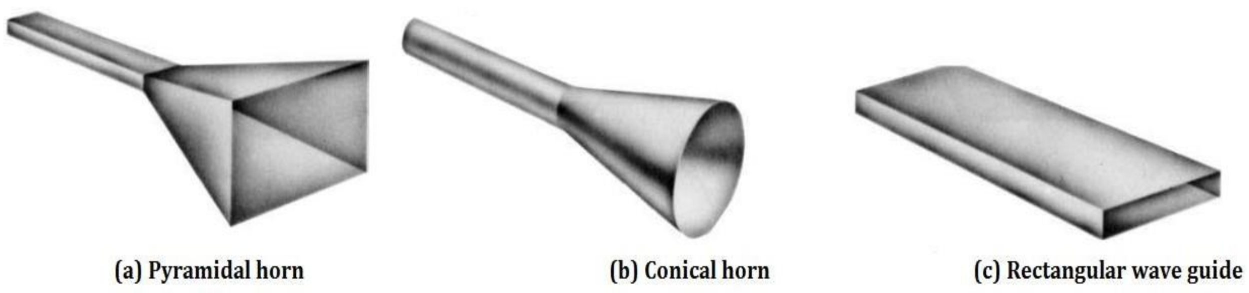


Fig. 2-1 Aperture antenna configurations

- Aperture antennas are very practical for space applications, because they can be flush mounted on the surface of the spacecraft or aircraft. Their opening can be covered with a dielectric material to protect them from environmental conditions. This type of mounting does not disturb the aerodynamic profile of the craft, which in high-speed applications is critical.
- The radiation characteristics of wire antennas can be determined once the current distribution on the wire is known. For many configurations, however the current distribution is not known exactly and only physical experimental measurements can provide a reasonable approximation to it. This is even more evident in aperture antennas. (Eg., slits, slots, waveguides, horns, reflectors, lenses).
- The idea used in the analysis of aperture type antennas is the conversion of the original antenna geometry into an equivalent geometry which can be looked at as radiation through an aperture in a closed surface. This equivalence is obtained by the principle known as field equivalence principle.
- Along with this principle, the duality, uniqueness theorem and image principles are also useful in the aperture type antenna analysis.

MAGNETIC CURRENT AND ITS FIELDS

- In wire antennas, the fields produced by the electric current distribution is computed via vector potential approach. The vector potential, $\boldsymbol{A}$, is the magnetic-type vector potential because the $\boldsymbol{H}$ field is directly related to the curl of $\boldsymbol{A}$. If the sources are radiating in free space , then ;

$$\boldsymbol{A}(x, y, z) = \frac{\mu}{4\pi} \iiint_{V'} \boldsymbol{J}(x', y', z') \; \frac{e^{-jkR}}{R} dv' \qquad \text{-------- (2.1)}$$

$$\boldsymbol{E} = -j\omega \boldsymbol{A} - \nabla V \; = -j\omega \boldsymbol{A} - \frac{j}{\omega\mu\epsilon} \nabla(\nabla . \boldsymbol{A}) \qquad \text{-------- (2.2)}$$

$$\boldsymbol{H} = \frac{1}{\mu} (\nabla \times \boldsymbol{A}) \qquad \text{-------- (2.3)}$$

- In the analysis of aperture antennas using the field equivalence principle, the concept of magnetic current density $\boldsymbol{M}$, is used.
- Let us assume that the flow of magnetic charges is the magnetic current which is similar to the concept that the flow of electric charges is the electric current.
- Similar to electric charge density ρ, magnetic charge density ρ , is used. With the introduction of magnetic charge density, Maxwell's Equations become ;

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + j\omega\epsilon \boldsymbol{E} \qquad \text{-------- (2.4)}$$

$$\nabla \times \boldsymbol{E} = -\boldsymbol{M} - j\omega\mu \boldsymbol{H} \qquad \text{-------- (2.5)}$$

$$\nabla . \boldsymbol{D} = \rho \qquad \text{-------- (2.6)}$$

$$\nabla . \boldsymbol{B} = \rho_m \qquad \text{-------- (2.7)}$$

$$\nabla . \boldsymbol{J} = -j\omega\rho \qquad \text{-------- (2.8)}$$

$$\nabla . \boldsymbol{M} = -j\omega\rho_m \qquad \text{-------- (2.9)}$$

- The magnetic charges postulated above do not exist in nature, but they are a useful mathematical construct to simplify the computation of fields. In the above equations if ρ_m and $\boldsymbol{M}$ are set to zero, we get back the equations in standard form.
- Let us define the $\boldsymbol{E}$ field as the curl of an electric vector potential , similar to $\boldsymbol{A}$.

$$\boldsymbol{E} = -\frac{1}{\epsilon} (\nabla \times \boldsymbol{F})$$

- The expression for $\boldsymbol{F}$ in terms of the magnetic current density, $\boldsymbol{M}$, in free space ;

$$\boldsymbol{F} = \frac{\epsilon}{4\pi} \iiint_{V'} \boldsymbol{M}(x', y', z') \; \frac{e^{-jkR}}{R} dv' \qquad \text{-------- (2.10)}$$

- The $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are related to $\boldsymbol{A}$, is related to the vector potential, $\boldsymbol{F}$;

$$\boldsymbol{E} = -\frac{1}{\epsilon} (\nabla \times \boldsymbol{F}) \qquad \text{-------- (2.11)}$$

$$\boldsymbol{H} = -j\omega \boldsymbol{F} - \frac{j}{\omega\mu\epsilon} \nabla(\nabla . \boldsymbol{F}) \qquad \text{-------- (2.12)}$$

- Thus, when both $\boldsymbol{J}$ and $\boldsymbol{M}$ are present, the fields are obtained by adding Eqns. (2.2) and (2.11) for the $\boldsymbol{E}$ field and Eqns. (2.3) and (2.12) for the $\boldsymbol{H}$ field ;

$$\boldsymbol{E} = -\frac{1}{\epsilon} (\nabla \times \boldsymbol{F}) - j\omega \boldsymbol{A} - \frac{j}{\omega\mu\epsilon} \nabla(\nabla . \boldsymbol{A}) \qquad \text{-------- (2.13)}$$

$$\boldsymbol{H} = \frac{1}{\mu} (\nabla \times \boldsymbol{A}) - j\omega \boldsymbol{F} - \frac{j}{\omega\mu\epsilon} \nabla(\nabla . \boldsymbol{F}) \qquad \text{-------- (2.14)}$$

UNIQUENESS THEOREM

- The uniqueness theorem states that for a given set of sources and the boundary conditions in a lossy medium, there exists a unique solution to the Maxwell's equations.
- Suppose in a isotropic, homogeneous medium, a source-free volume V is bounded by a surface , and if ($\boldsymbol{E}_1$, $\boldsymbol{H}_1$) are the fields inside the volume V produced by external sources and if ($\boldsymbol{E}_2$, $\boldsymbol{H}_2$) is another set of fields in the volume V produced by same external sources, then it can be proved that the fields are identical everywhere in the volume V , if either the tangential component of $\boldsymbol{E}$ field or tangential component of $\boldsymbol{H}$ field is same on the surface S.
- The solution to the Maxwell's equations to be unique, only the tangential component of $\boldsymbol{E}$ field or tangential component of $\boldsymbol{H}$ field is to be equated on the surface S.
- So, in a source free region, the fields are completely determined by the tangential component of $\boldsymbol{E}$ field or tangential component of $\boldsymbol{H}$ field on the surface S.

FIELD EQUIVALENCE PRINCIPLE

⇨ Huygen's Principle:

- The field equivalence was introduced in 1926 by S. A. Schelkunoff and it is a more rigorous formulation of Huygens' principle which states that "*each point on a primary wavefront can be considered to be a new source of a secondary spherical wave and that a secondary wavefront can be constructed as the envelope of these secondary spherical waves.*"
- *The field equivqlence principle is a principle by which actual sources, such as antenna and transmitter, are replaced by equivalent sources. The fictitious sources are said to be equivalent within a region because they produce the same fields within that region.*
- By the equivalence principle, the fields outside an imaginary closed surface are obtained by placing over the closed surface suitable electric and magnetic-current densities which satisfy the boundary conditions. The current densities are selected so that the fields inside the closed surface are zero and outside they are equal to the radiation produced by the actual sources.
- The equivalence principle is developed by considering an actual radiating source, which electrically is represented by current densities $\boldsymbol{J}_1$ and $\boldsymbol{M}_1$, as shown in Fig. 2-2(a). The source radiates fields $\boldsymbol{E}_1$and $\boldsymbol{H}_1$everywhere.
- However, it is desired to develop a method that will yield the fields outside a closed surface. To accomplish this, a closed surface S is chosen, shown dashed in Fig. 2-2 (a), which encloses the current densities $\boldsymbol{J}_1$ and $\boldsymbol{M}_1$. The volume within S is denoted by $V_1$and outside S by V_2.
- The primary task will be to replace the original problem, shown in Fig. 2-2(a), by an equivalent one which yields the same fields $\boldsymbol{E}_1$and $\boldsymbol{H}_1$ outside S (within V_2).
- An equivalent problem of Fig. 2-2 (a), is shown in Fig. 2-2 (b). The original sources $\boldsymbol{J}_1$ and $\boldsymbol{M}_1$ are removed, and we assume that there exist fields $\boldsymbol{E}$ and $\boldsymbol{H}$ inside S and fields $\boldsymbol{E}_1$and $\boldsymbol{H}_1$ outside of S. For these fields to exist within and outside S, they must satisfy the boundary conditions on the tangential electric and magnetic field components. Thus on the imaginary surface S there must exist the equivalent sources and they radiate into an unbounded space (same medium everywhere).

$$J_s = \hat{n} \times [H_1 - H] \qquad \text{-------- (2.15)}$$

$$M_s = -\hat{n} \times [E_1 - E] \qquad \text{-------- (2.16)}$$

- The current densities of Eqn. (2.15) and (2.16) are said to be equivalent only within V_2, because they produce the original fields ($\boldsymbol{E_1}$, $\boldsymbol{H_1}$) only outside S. Fields $\boldsymbol{E}$, $\boldsymbol{H}$ different from the originals ($\boldsymbol{E_1}$, $\boldsymbol{H_1}$), result within V_1.

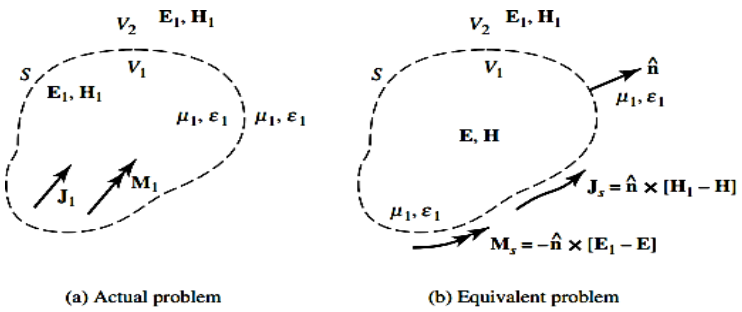


Fig. 2-2. Actual and equivalent models.

- From electromagnetic uniqueness concepts, it is known that the tangential components of only $\boldsymbol{E}$ or $\boldsymbol{H}$ are needed to determine the fields. Since the fields $\boldsymbol{E}$, $\boldsymbol{H}$ within S can be anything, it canbe assumed that they are zero.

- In that case the equivalent problem of Fig. 2-2 (b) reduces to that of Fig. 2-2 (a) with the equivalent current densities being equal to ;

$$J_s = \hat{n} \times [H_1 - H]|_{H=0} = \hat{n} \times H_1 \qquad \text{-------- (2.17)}$$

$$M_s = -\hat{n} \times [E_1 - E]|_{E=0} = -\hat{n} \times E_1 \qquad \text{-------- (2.18)}$$

- This form of the field equivalence principle is known as Love's Equivalence Principle.

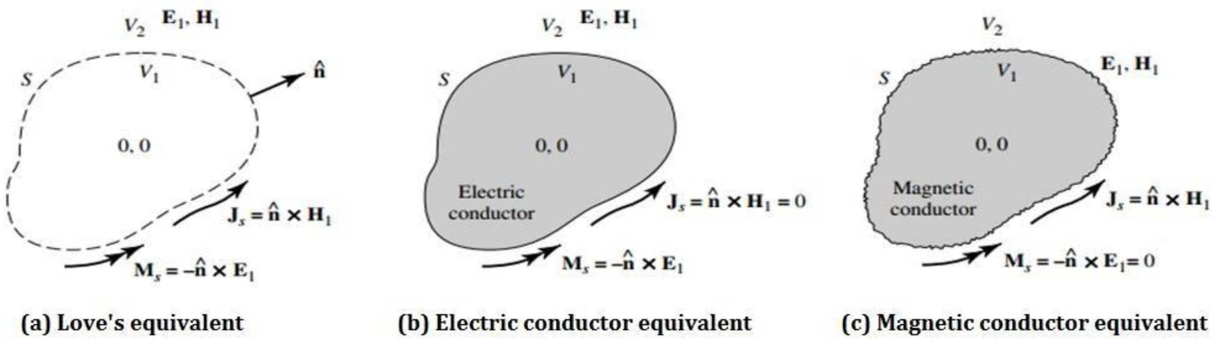


Fig. 2-2 Equivalence principle models

- Love's Equivalence Principle of Fig. 2-3 (a) produces a null field within the imaginary surface S. Since the value of the $\boldsymbol{E} = \boldsymbol{H} = \boldsymbol{0}$ within S cannot be disturbed if the properties of the medium within it are changed, let us assume that it is replaced by a perfect electric conductor ($\sigma = \infty$).

- The introduction of the perfect conductor will have an effect on the equivalent source $\boldsymbol{J_s}$. As the electric conductor takes its place, as shown in Fig. 2-3 (b), the electric current density $\boldsymbol{J_s}$, which is tangent to the surface S, is short-circuited by the electric conductor. Thus the equivalent problem of Fig. 2-3 (a) reduces to that of Fig. 2-3 (b). There exists only a magnetic current density $\boldsymbol{M_s}$ over S, and it radiates in the presence of the electric conductor producing outside S the original fields $\boldsymbol{E_1}$, $\boldsymbol{H_1}$.

- Let us assume that instead of placing a perfect electrical conductor within S we introduce a perfect magnetic conductor which will short out the magnetic current density and reduce the equivalent problem to that shown in Fig. 2-3 (c).

Summary:

The steps that must be used to form an equivalent and solve an aperture problem are as follows:

- Select an imaginary surface that encloses the actual sources (the aperture). The surface must be chosen so that the tangential components of the electric and/or the magnetic field are known, exactly or approximately, over its entire span. In many cases this surface is a flat plane extending to infinity.
- Over the imaginary surface form equivalent current densities $\boldsymbol{J_s}$, $\boldsymbol{M_s}$ which take one of the following forms:

  a. $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$ over S assuming that the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields within S are not zero.
  b. or $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$ over S assuming that the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields within S are zero (Love's theorem)
  c. or $\boldsymbol{M_s}$ over S ($\boldsymbol{J_s} = 0$) assuming that within S the medium is a perfect electric conductor
  d. or $\boldsymbol{J_s}$ over S ($\boldsymbol{M_s} = 0$) assuming that within S the medium is a perfect magnetic conductor.
- Solve the equivalent problem and then compute the field components.

Example:

A waveguide aperture is mounted on an infinite ground plane, as shown in Fig. 2-4 (a). Assuming that the tangential components of the electric field over the aperture are known, and are given by $\boldsymbol{E_a}$, find an equivalent problem that will yield the same fields $\boldsymbol{E}$, $\boldsymbol{H}$ radiated by the aperture to the right side of the interface.

Solution:

First an imaginary closed surface is chosen. For this problem it is appropriate to select a flat plane extending from minus infinity to plus infinity, as shown in Fig. 2-4 (b). Over the infinite plane, the equivalent current densities $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$ are formed. Since the tangential components of $\boldsymbol{E}$ do not exist outside the aperture, because of vanishing boundary conditions, the magnetic current density $\boldsymbol{M_s}$ is only non-zero over the aperture. The electric current density $\boldsymbol{J_s}$ is non-zero everywhere and is yet unknown. Now let us assume that an imaginary flat electric conductor approaches the surface S, and it shorts out the current density $\boldsymbol{J_s}$everywhere. $\boldsymbol{M_s}$ exists only over the space occupied originally by the aperture, and it radiates in the presence of the conductor [Fig. 2-4 (c)]. By image theory, the conductor can be removed and replaced by an imaginary (equivalent) source $\boldsymbol{M_s}$ as shown in Fig. 2-4 (d), which is analogous to Fig. 2-4(b). Finally, the equivalent problem of Fig 2-4 (d) reduces to that of Fig. 2-4(e), which is analogous to that of Fig. 2-4 (c).

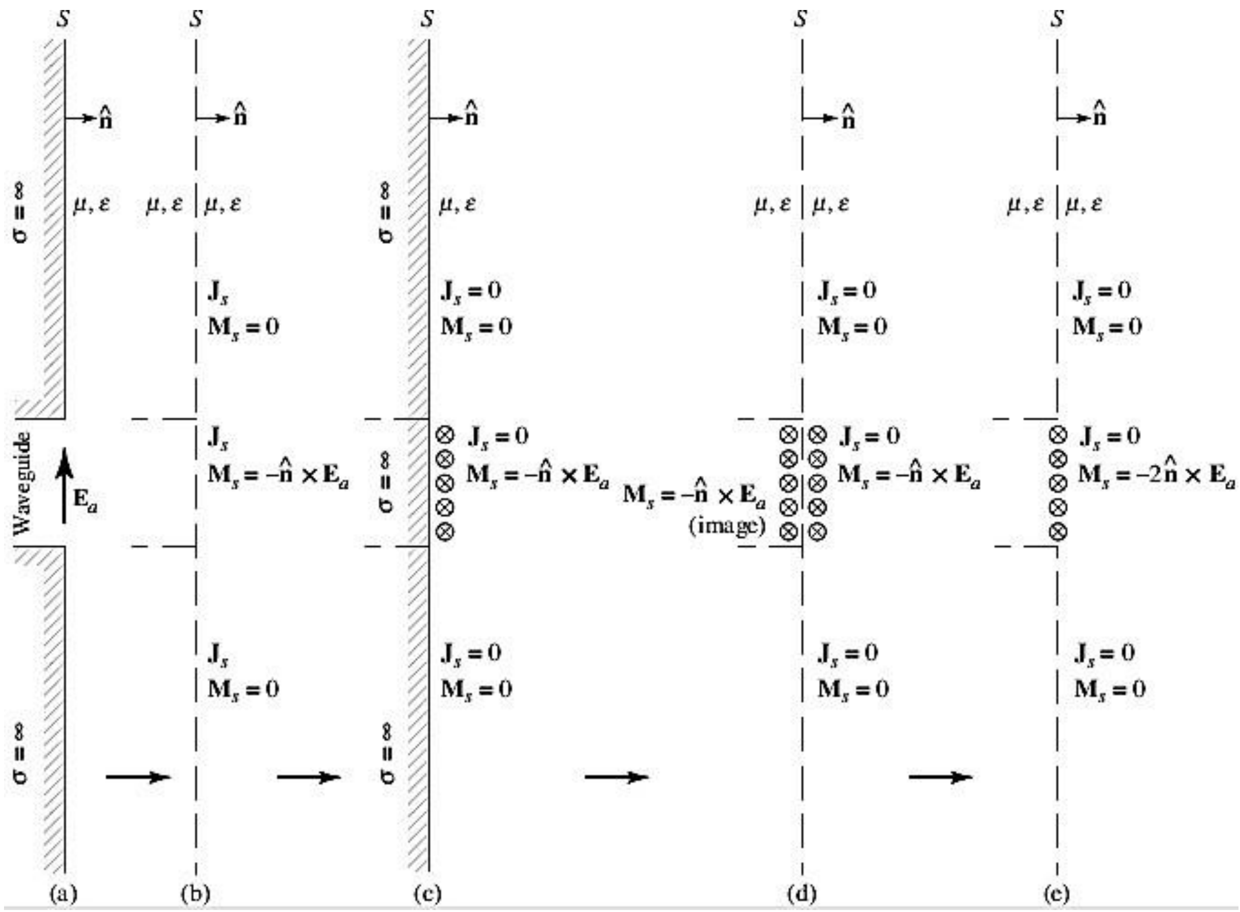


Fig. 2-4 Equivalent models for waveguide aperture mounted on an infinite flat electric ground plane.

RADIATION FRPM APERTURE FIELDS

- The general coordinate system for aperture antenna analysis is shown in Fig. 2-5. The radiating fields are determined by first finding the vector potentials $\boldsymbol{A}$ and , from the surface current densities $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$ respectively.

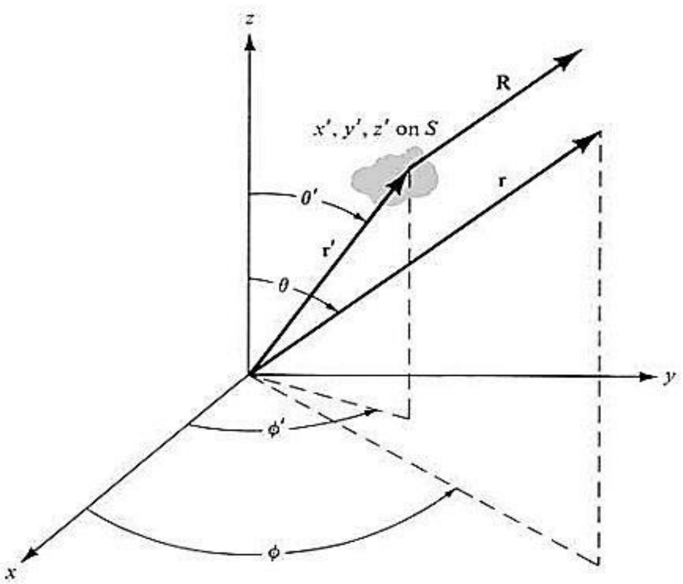


Fig. 2-5 Coordinate system for aperture antenna analysis.

- For far-field approximation:

$$R \approx r - r'\cos\psi \qquad \text{for phase variations} \qquad \text{-------- (2.19)}$$

$$R \approx r \qquad \text{for amplitude variations} \qquad \text{-------- (2.20)}$$

- The auxiliary potential functions $\boldsymbol{A}$ and $\boldsymbol{F}$ generated by the current densities $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$ is given by ;

$$\mathbf{A} = \frac{\mu}{4\pi}\iint_S \mathbf{J}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\mu e^{-jkr}}{4\pi r}\mathbf{N} \qquad \text{-------- (2.21)}$$

$$\mathbf{N} = \iint_S \mathbf{J}_s e^{jkr'\cos\psi}\, ds' \qquad \text{-------- (2.22)}$$

$$\mathbf{F} = \frac{\epsilon}{4\pi}\iint_S \mathbf{M}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\epsilon e^{-jkr}}{4\pi r}\mathbf{L} \qquad \text{-------- (2.23)}$$

$$\mathbf{L} = \iint_S \mathbf{M}_s e^{jkr'\cos\psi}\, ds' \qquad \text{-------- (2.24)}$$

- In the far-field only the θ and ϕ components of the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are dominant.

$$\begin{aligned} (E_A)_\theta &= -j\omega A_\theta \\ (E_A)_\phi &= -j\omega A_\phi \\ (E_F)_\theta &= -j\omega \eta F_\phi \\ (E_F)_\phi &= j\omega \eta F_\theta \end{aligned} \qquad \text{-------- (2.25)}$$

$$\begin{aligned} (H_A)_\theta &= j\omega \frac{A_\phi}{\eta} \\ (H_A)_\phi &= -j\omega \frac{A_\theta}{\eta} \\ (H_F)_\theta &= -j\omega F_\theta \\ (H_F)_\phi &= j\omega F_\phi \end{aligned} \qquad \text{-------- (2.26)}$$

- Combining Eqn's. (2.25) to (2.26), yields :

$$\begin{aligned} E_r &\approx 0 \\ E_\theta &\approx -j\frac{\kappa}{4\pi r}[L_\phi + nN_\theta]e^{-j\kappa r} \\ E_\phi &\approx j\frac{\kappa}{4\pi r}[L_\theta - \eta N_\phi]e^{-j\kappa r} \\ H_r &\approx 0 \\ H_\theta &\approx j\frac{\kappa}{4\pi r}[N_\phi - \frac{L_\theta}{\eta}]e^{-j\kappa r} \\ H_\phi &\approx -j\frac{\kappa}{4\pi r}[N_\theta + \frac{L_\phi}{\eta}]e^{-j\kappa r} \end{aligned} \qquad \text{-------- (2.27)}$$

where ;

$$N_\theta = \iint_S \left[J_x \cos\theta\cos\varphi + J_y \cos\theta\sin\varphi - J_z \sin\theta\right] e^{+jkr'\cos\psi} ds'$$
$$N_\varphi = \iint_S \left[-J_x \sin\varphi + J_y \cos\varphi\right] e^{+jkr'\cos\psi} ds'$$
$$L_\theta = \iint_S \left[M_x \cos\theta\cos\varphi + M_y \cos\theta\sin\varphi - M_z \sin\theta\right] e^{+jkr'\cos\psi} ds'$$
$$L_\varphi = \iint_S \left[-M_x \sin\varphi + M_y \cos\varphi\right] e^{+jkr'\cos\psi} ds'$$

-------- (2.28)

RADIATION FROM RECTANGULAR APERTURES.

A. Uniform Aperture:

- Consider that a rectangular aperture is mounted on an infinite ground plane (Fig. 2-6) assuming field over the opening to be constant and is given by :

$$\mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \quad -a/2 \le x' \le a/2, \quad -b/2 \le y' \le b/2$$

-------- (2.28)

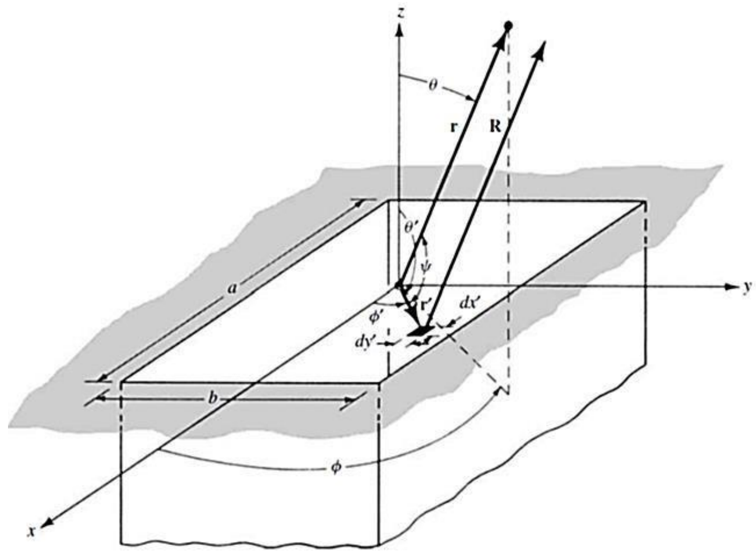


Fig. 2-6 Rectangular aperture on an infinite electric ground plane.

- To obtain equivalent of this situation, assuming a closed surface extending from $-\infty$ to ∞ on the xy plane. Then we can write:

$$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a = -2\hat{\mathbf{a}}_z \times \hat{\mathbf{a}}_y E_0 = +\hat{\mathbf{a}}_x 2E_0 & \begin{matrix} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{matrix} \\ 0 & \text{elsewhere} \end{cases}$$
$$\mathbf{J}_s = 0 \qquad \text{everywhere}$$

-------- (2.29)

- The far-zone fields radiated by the aperture of Fig. 2-6 can be found by using Eqn. (2.27), (2.28) and (2.9). Thus,

$$N_\theta = N_\phi = 0$$
$$L_\theta = \int_{-b/2}^{+b/2} \int_{-a/2}^{+a/2} [M_x \cos\theta\cos\phi] e^{jk(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi)} dx'\, dy'$$

-------- (2.30)

$$\boxed{L_\theta = \cos\theta\ \cos\phi \left[\int_{-b/2}^{+b/2}\int_{-a/2}^{+a/2} M_x e^{jk(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi)}\, dx'\, dy'\right]}$$ -------- (2.31)

Using the integral

$$\int_{-c/2}^{+c/2} e^{j\alpha z}\, dz = c\left[\frac{\sin\left(\frac{\alpha}{2}c\right)}{\frac{\alpha}{2}c}\right]$$

- Eqn. (2.31) reduces to ;

$$L_\theta = 2abE_0\left[\cos\theta\cos\phi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$

where

$$X = \frac{ka}{2}\sin\theta\cos\phi$$ -------- (2.32)

$$Y = \frac{kb}{2}\sin\theta\sin\phi$$

- Similarly ;

$$L_\phi = -2abE_0\left[\sin\phi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$ -------- (2.33)

- From the above Eqn's. the fields radiated by the aperture can be written as

$$E_r = 0$$
$$E_\theta = j\frac{abkE_0e^{-jkr}}{2\pi r}\left[\sin\phi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$
$$E_\phi = j\frac{abkE_0e^{-jkr}}{2\pi r}\left[\cos\theta\cos\phi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$
$$H_r = 0$$
$$H_\theta = -\frac{E_\phi}{\eta}$$
$$H_\phi = +\frac{E_\theta}{\eta}$$ -------- (2.34)

E – Plane ($\phi = \pi/2$)

$$E_r = E_\phi = 0$$

$$E_\theta = j\frac{abkE_0e^{-jkr}}{2\pi r}\left[\frac{\sin\left(\frac{kb}{2}\sin\theta\right)}{\frac{kb}{2}\sin\theta}\right]$$

H –Plane ($\phi = 0$)

$$E_r = E_\theta = 0$$

$$E_\phi = j\frac{abkE_0e^{-jkr}}{2\pi r}\left\{\cos\theta\left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta}\right]\right\}$$

B. Tapered Aperture:

- One practical aperture of tapered source distribution is the open rectangular waveguide. The dominant TE_{10} mode has the following distribution:

$$\mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \cos\left(\frac{\pi}{a}x'\right)\left\}\begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array}\right. \quad \text{-------- (2.35)}$$

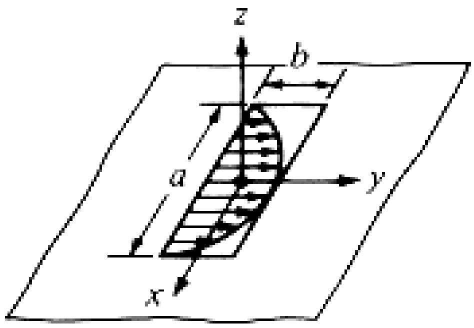


Fig. 2-7 TE_{10} -Mode Distribution Aperture on Ground Plane.

- To obtain equivalent of this situation, assuming a closed surface extending from $-\infty$ to ∞ on the xy plane. Then we can write:

$$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a & \left.\begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array}\right. \\ 0 & \text{elsewhere} \end{cases}$$
$$\mathbf{J}_s = 0 \qquad \text{everywhere} \quad \text{-------- (2.36)}$$

- Similarly the fields are computed as ;

$$E_r = H_r = 0$$
$$E_\theta = -\frac{\pi}{2}C\sin\phi\frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2}\frac{\sin Y}{Y}$$
$$E_\phi = -\frac{\pi}{2}C\cos\theta\cos\phi\frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2}\frac{\sin Y}{Y} \quad \text{-------- (2.37)}$$
$$H_\theta = -E_\phi/\eta$$
$$H_\phi = E_\theta/\eta$$

SLOT ANTENNA

- Slot antenna is a radiating element formed by a slot in a metallic surface. An opening cut in a conducting sheet or in one of the walls of the waveguide acts as the antenna. It is excited suitably either by a co-axial cable or through the waveguide. Slot antenna is the best suitable radiator at frequencies above 200 MHz.
- Consider an infinite conducting sheet as in Fig. 2-8 (a). Now consider that an aperture of any size or shape is made leaving a slot on a sheet. The flat strip obtained can be treated as short dipole as shown in Fig. 2-8 (b).

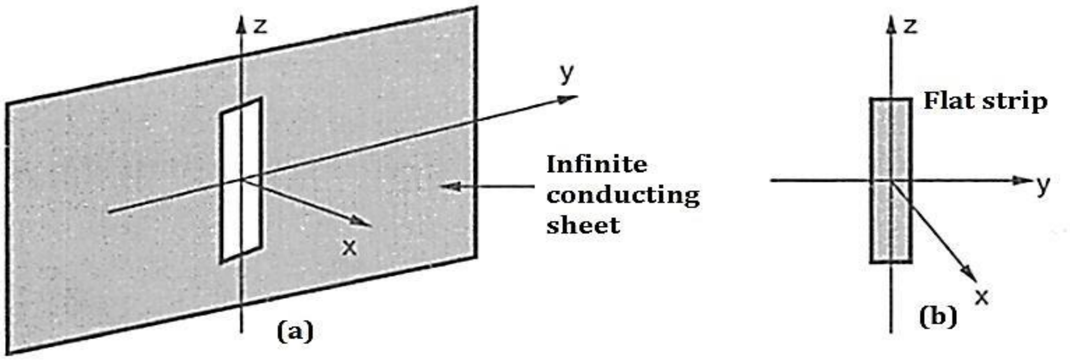


Fig. 2-8 Metallic conducting sheet and complementary flat strip

- When the two are combined together we get the complete original infinite conducting sheet. Hence the infinite conducting sheet with slot and the flat strip of the dimension same as of the slot are said to be complementary.
- Now consider that a slot of $\lambda/2$ is cut in a large conducting sheet, we get complementary dipole antenna. In general, the slot antenna is fed by either a generator (or) transmission line connected across it.
- **Principle:** *Whenever a high frequency field exists across a very narrow slot in an infinite conducting sheet, the energy is radiated through slot. This is the working principle of the slot antenna.*
- In case of the waveguides, the slot antenna is fed with the guided waves incident on slot. Consider that the slot antenna is fed with a transmission line connected across points A & B as in Fig. 2-9 (a).
- As the antenna is fed with a transmission line, the slot will radiate due to the currents in the conducting sheet. The complementary of the slot antenna is the dipole as shown in Fig. 2-9 (b).
- For the complementary dipole antenna, the regions with conducting sheet and air are interchanged. According to the G. Booker's theory, the field pattern of the slot is exactly identical in shape as that of the half dipole with $\boldsymbol{E}$ and $\boldsymbol{H}$ interchanged. That means for the slot, the electric field $\boldsymbol{E}$ will be horizontally polarized, while for the dipole, it is vertically polarized.
- A single half wavelength slot in a conducting sheet is analogous to the half wave dipole in terms of gain and directivity with only difference in the polarization. The horizontal slot

produces vertical polarization in the direction normal to the slot, while the vertical slot produces horizontal polarization.

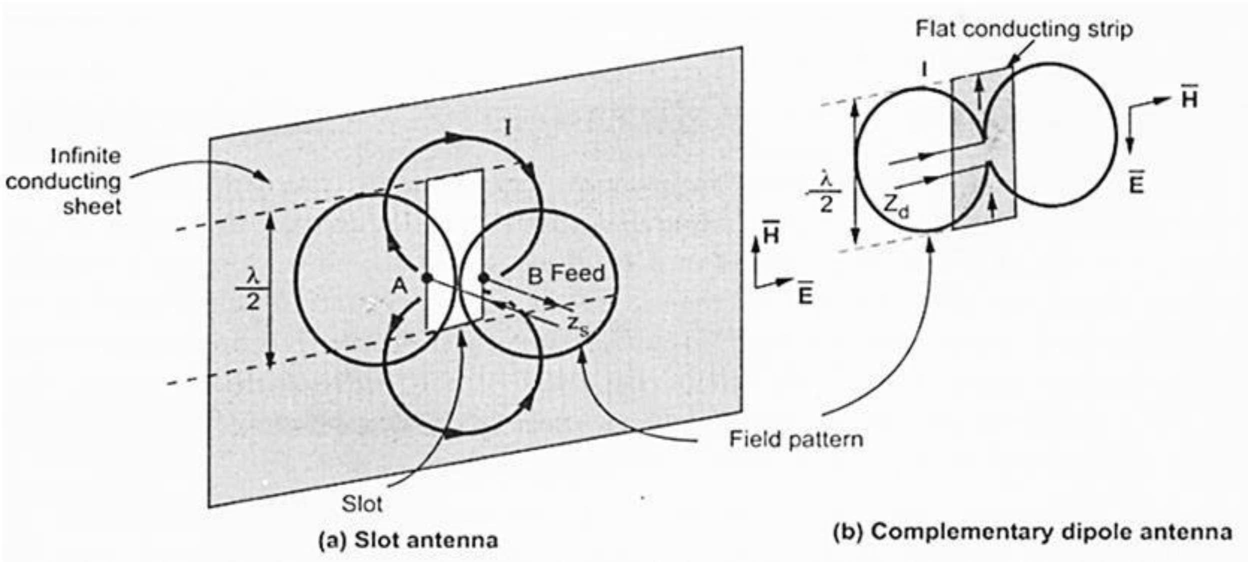


Fig. 2-9 Slot and complementary dipole antenna.

- Although the width of the slot is small ($\omega \ll \lambda$), the currents are not confined to the edges of the slot but spread out over the sheet.
- The terminal impedance Z_s of the slot is related to the terminal impedance of dipole Z_d by intrinsic impedance η of free space by the relation,

$$Z_s Z_d = \frac{\eta^2}{4} = \frac{(277)^2}{4} \qquad \text{-------- (2.38)}$$

$$Z_s \simeq \frac{25{,}476}{Z_d} \qquad \text{-------- (2.39)}$$

- Suppose that the terminal impedance of dipole antenna is $Z_d = 72 + j\,42.5\ \Omega$, then the terminal impedance of the complementary slot will be,

$$Z_s = \frac{25476}{72 + j\,42.5} = \frac{25476 \times (72 - j\,42.5)}{(72 + j\,42.5)\,(72 - j\,42.5)}$$

$$Z_s \approx 262 - j\,211\ \Omega \qquad \text{-------- (2.40)}$$

- The differences between the slot antenna and its complementary dipole antenna are,
    - Polarization is different in both the antennas. That means if the polarization is horizontal in slot antenna, then it is vertical in the complementary antenna.
    - The radiations from the backside of the slot antenna and the complementary antenna are of opposite polarity.

⇨ **Method of Feeding Slot Antennas:**

- Practically the slot antenna is fed with a co-axial transmission line. The outer conductor is bonded to the metal sheet as shown in Fig. 2-10 (a). In general, the terminal impedance of λ/2.
- Slot in a conducting large sheet is very large (approximately 500 Ω), while the characteristic impedance of the transmission line is much smaller. Thus under such conditions, to provide proper impedance matching, the off-center feed as shown in Fig. 2-10 (b) is used.

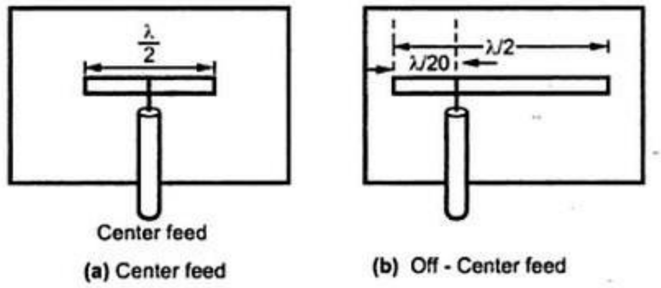


Fig. 2-10 Feeding of slot antenna using co-axial cable.

- In practical applications, generally a slot antenna fed with a transmission line is not observed. Instead of that the slot is boxed in any one of the sides of the metallic cavity so that by properly selecting dimensions of the cavity, the outward radiation from the opening of the cavity is not affected, while the backward radiation is virtually eliminated.
- For the applications at very high frequencies, a slot antenna with a slot cut in a conducting cylinder is most widely used. A longitudinal slot in infinitely long cylinder as shown in Fig. 2-11 (a), produces circular radiation that diameter of the cylinder is very small.
- The gain and directivity properties of a basic slot antenna can be improved by using array of slots placed half guide wavelength apart and placed on opposite side of central line as shown in Fig. 2-11 (b). Actually are the slots radiates in same phase, but there is a reversal of polarity of the field inside the guide. This is compensated by placing slots alternately on opposite sides of the central line.

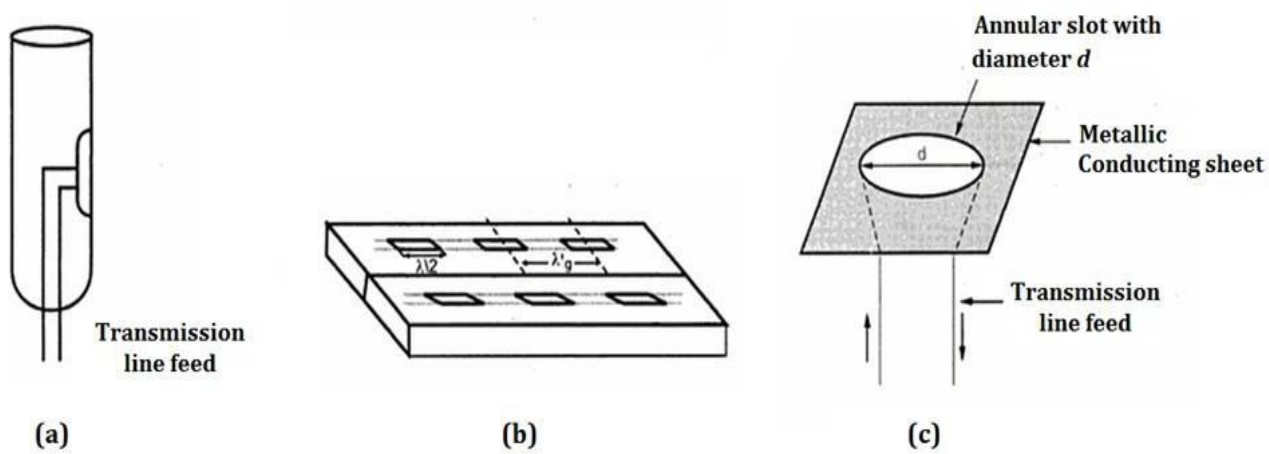


Fig. 2-11 a) Cylindrical slot antenna b) Planar array slot antenna c) Annular slot antenna

- The shape of the slot may be either rectangular or circular. The slot with circular or annular shape is called annular slot antenna. The annular slot antenna is shown in 2-11 (c). When the diameter of the annular slot is less than half wavelength the resulting radiations are identical to those produced by short, vertical antenna.

- **Boxed In Slot Antenna**: A flat sheet with a $\lambda/2$ slot radiates equally on both sides of the sheet. However, if the sheet is very large (ideally infinite) and boxed in as in Fig. 2-12, radiation occurs only from one side. The depth d of the box is approximate ($d \sim \lambda/4$) for a thin slot.

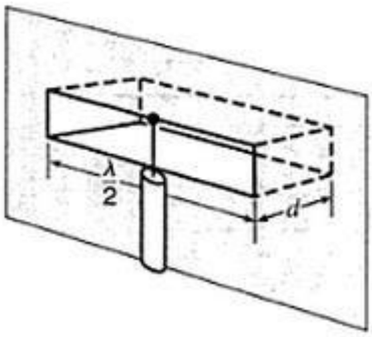


Fig. 2-12 Boxed-in slot antenna

⇨ **Babinet's Principle:**

- Babinet's principle states that when the field behind a screen with an opening is added to the field of a complementary structure, the sum is equal to the field when there is no screen.

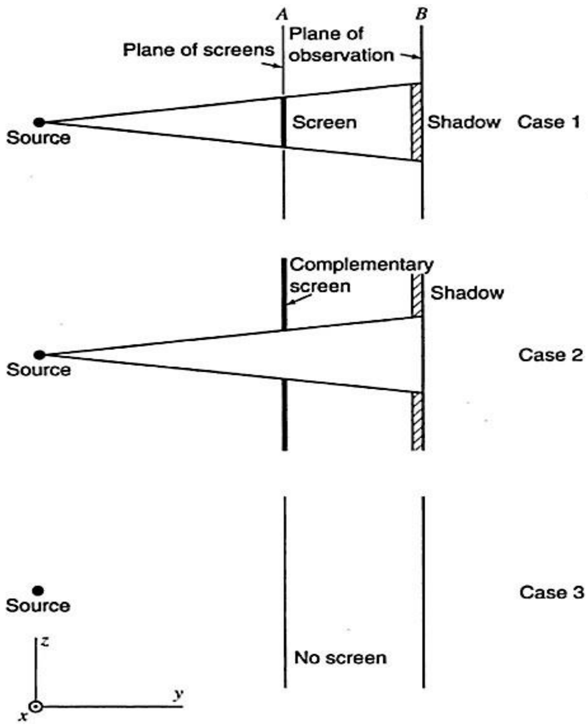


Fig. 2-13 Illustartion of Babinet's principle.

- Consider case 1 where a perfectly absorbing screen is placed in plane A which has a region of shadow in the observation plane B as shown in Fig. 2-13 (a).
- Let the field behind the screen be some function f_1 of x, y and z is given by $F_s = f_1(x, y, z)$.

- Now case 2 where the first screen is replaced by its complementary screen as shown in Fig. 2-13 (b).
- Let the field behind complementary screen is given by $F_{cs} = f_2(x, y, z)$.
- Finally in case 2, no screen is present and the field is given by $F_0 = f_2(x, y, z)$.
- Now according to Babinet's principle, at same point, the toatal field is given as $F_0 = F_s + F_{cs}$.

HORN ANTENNA

- One of the simplest and probably the most widely used microwave antenna is the horn. The horn is widely used as a feed element for large radio astronomy, communication dishes and satellite tracking through out the world.
- The horn antenna can be considered as a a waveguide with hollow pipe of different cross-sections which is flared or tapered into a large opening. When one end of the waveguide is excited while other end is kept open, it radiates in open space in all directions.
- As compared with the radiation through transmission line, the radiation through the waveguide is larger. In waveguide, a small portion of the incident wave is radiated and large portion is reflected back due to the open circuit.
- As one end of the waveguide is open circuited, the impedance matching with the free space is not perfect. To minimize reflections of the guided wave , the mouth of the waveguide is flared or opened out such that it assumes shape like horn.
- A horn antenna is nothing but a flared out or opened out waveguide. The main function of the horn antenna is to produce an uniform phase front with a aperture larger than waveguide to give higher directivity.

⇨ Types of Horn Antennas

- Basically the horn antennas are classified as rectangular horn antennas and circular horn antennas. The rectangular horn antennas are fed with rectangular waveguide, while the circular horn antennas are fed with circular waveguide.
- Depending upon the direction of flaring, the rectangular horns are further classified as Sectoral horn and Pyramidal horn. A sectoral horn is obtained if the flaring (tapering) is done in one direction only. A sectoral horn is further classified as E-plane sectoral horn and H-plane sectoral horn.
- The E-plane sectoral horn is obtained when the flaring is done in the direction of the electric field vector. The H-plane sectoral horn is obtained if the flaring is done in the direction of the magnetic field vector.
- When the flaring is done along both the walls of the rectangular waveguide in the direction of both the electric and magnetic field vectors, the horn obtained is called pyramidal horn.
- Similar to the rectangular horns, the circular horn antennas can be obtained by flaring the walls of the circular waveguide. The circular horn antennas are of two types namely conical horn antenna and biconical horn antenna.

- Many times, the transition region between the throat of the waveguide and the aperture is tapered with a gradual exponential taper. This minimizes the reflections of the guided waves. Such horns are called exponentially tapered horn antennas.
- Fig. 2-14 shows the horn antennas such as the E-plane sectoral horn, H-plane sectoral horn, pyramidal horn and conical horn.
- When the aperture size is large compared to the wavelength the wave impedance approaches the free space impedance, asymptotically. Thus, a pyramidal horn provides a slow transition from the waveguide impedance to the free space impedance, provided that the length of the transition is large compared to the wavelength.

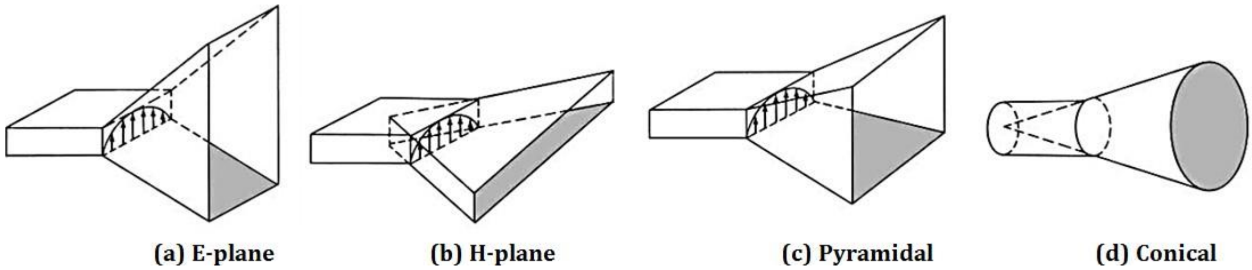


Fig. 2-14 Typical horn antennas

⇨ **Design of Horn antenna:**

- The cross sectional view of rectangular horn antenna is shown in Fig. 2-15.

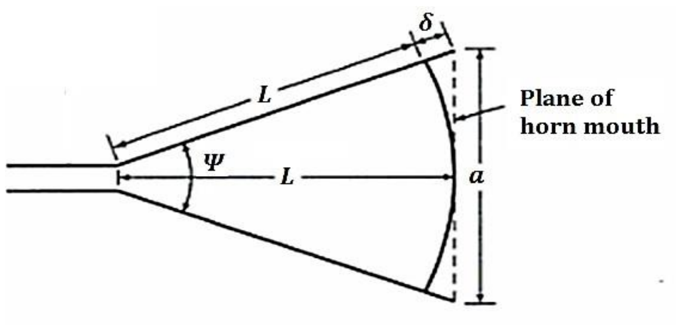


Fig. 2-15 Cross sectional view of rectangular horn antenna

From the geometry ;

$$\cos\frac{\Psi}{2} = \frac{L}{L+\delta}$$

$$\sin\frac{\Psi}{2} = \frac{a}{2(L+\delta)}$$

$$\tan\frac{\Psi}{2} = \frac{a}{2L}$$

where Ψ = flare angle (Ψ_E for E plane, Ψ_H for H plane), a = aperture (a_E for E plane, a_H for H plane) , L = horn length and δ = path difference.

- From the geometry we have also that

$$L = \frac{a^2}{8\delta} \qquad (\delta \ll L) \qquad \text{-------- (2.41)}$$

$$\Psi = 2\tan^{-1}\frac{a}{2L} = 2\cos^{-1}\frac{L}{L+\delta} \qquad \text{-------- (2.42)}$$

- When the flare angle is small, the aperture area for a specified length becomes small. Thus at the mouth of the horn, the uniform phase front is resulted, which increases directivity with decrease in the beam width. The angle represented in Eqn. (2.42) is known as optimum aperture angle.

- The directivity of maximum value can be obtained at the largest flare angle for which the value δ does not exceed typical value such as 0.25 λ for E-plane sectoralhorn, 0.22 λ for conical horn and 0.40 λ for H-plane sectoral horn.
- The directivity of the pyramidal horn and conical horn is highest as compared to other types of the horns because they have more than one flare angle. One more advantage of the horn antenna is that it can be operated over a wide range of high frequency as there is no resonant element in the antennna.
- **Optimum Horn:** Generally to obtain uniform aperture distribution, it is observed that the horn should be very long with small flare angle. But for practical convenience, the horn should be as short as possible. Betweeen these two extreme conditions, it is possible to design a horn which has minimum beamwidth and for a given length it is free of side lobes. Such a horn is called optimum horn or optium flare horn.
- Hence, for optimum dimension δ such that for largest flare angle directivity is maximum and such optimum value is δ_0. The dimensions of optimum horn as follows;

Optimum value of δ: $$\delta_0 = \frac{L}{\cos\frac{\Psi}{2}} - L \quad \text{-------- (2.43)}$$

Optimum length L: $$L = \frac{\delta_0 \cos\frac{\Psi}{2}}{1 - \cos\frac{\Psi}{2}} \quad \text{-------- (2.44)}$$

- For optimum flare horn, the HPBW is approximated as ;

$$\Psi_H = \frac{67°\lambda}{a_H} \quad \text{and} \quad \Psi_E = \frac{56°\lambda}{a_E} \quad \text{-------- (2.45)}$$

- FNBW is approximated as ;

$$\Psi_H = \frac{172°\lambda}{a_H} \quad \text{and} \quad \Psi_E = \frac{115°\lambda}{a_E} \quad \text{-------- (2.46)}$$

- Directive gain $(D)$and power gain (G) in terms of effective effecture of the horn as ;

$$\frac{4\pi A_e}{\lambda^2} \quad \frac{4\pi \varepsilon_{ap} A_p}{\lambda^2} \quad \frac{7.5 A_p}{\lambda^2} \qquad [\because \varepsilon_{ap} \approx 0.6] \quad \text{-------- (2.47)}$$

$$\frac{4.5 A_p}{\lambda^2} \quad \text{-------- (2.48)}$$

where ; A_e = Effective aperture , A_p = Physical aperture and ε_{ap} = Aperture efficiency.

- For rectangular horn ; $A_p = a_E.\, a_H$ and For conical horn ; $A_p = \pi r^2$

⇨ <u>**Applications of horn**</u>

- It is used as a feed element in antennas such as parabolic reflectors
- It is the most wide used antenna for measurement of various antenna parameters in the laboratories.
- It is most suitable antenna for various application in microwave frequency range where moderate gains are sufficient.

REFLECTOR ANTENNAS

- The reflector antennas are most important in microwave radiation applications. At microwave frequencies the physical size of the high gain antenna becomes so small that practically any suitable shaped reflector can produce desired directivity.
- In reflector antenna, another antenna is required to excite it. Hence the antenna such as dipole, horn, slot which excites the reflector antenna is called primary antenna, while the reflector antenna is called secondary antenna.
- In general, reflector antenna can be represented in any geometrical configuration, but the most commonly used shapes are plane reflector, corner reflector and curved or parabolic reflectors. Using reflectors, the radiation pattern of a radiating antenna can be modified.
- By using a large, metallic plane sheet as a reflector, the backward radiations from the antenna can be eliminated thus improving radiation pattern of an antenna. Thus for an antenna, desired radiation characteristics can be produced with the help of a large, suitably illuminated and suitably sized and shaped reflector surface. Some of the common reflectors are plane reflector, corner reflector and curved reflector.

⇨ Flat Reflector / Plane refelector:

- The plane reflector is the simplest form of the reflector antenna. When the plane reflector is kept infront of the feed, the energy is radiated in the desired direction. The plane reflector is as shown in Fig. 2-16.

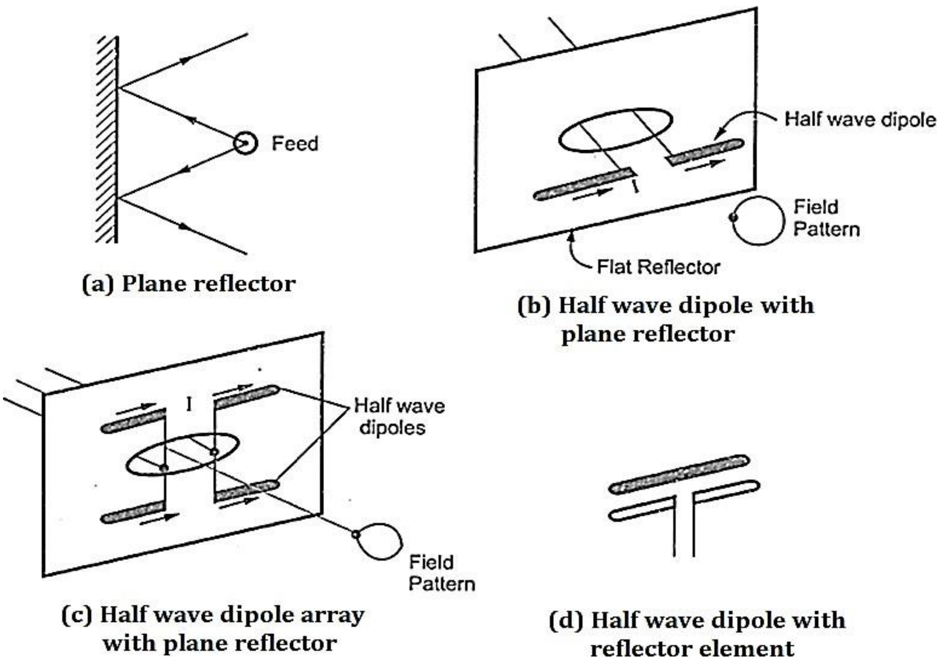


Fig. 2-16 Plane reflector and examples

- To increase the directivity of the antenna, a large flat sheet can be kept as plane reflector infront of a half dipole as shown in Fig. 2-16 (b).
- The main advantage of the plane reflector is that for the dipole backward radiations are reduced and the gain in the forward direction increases. To increase directivity further, we can use array of two half wave dipoles infront of a flat plane reflector as shown in Fig. 2-16 (c).

- It is observed that the flat sheet is less frequency sensitive than the thin element. Hence only a reflector element can be used to increase directivity. Such arrangement is shown in Fig. 2-16 (d).
- In case of the plane reflectors, the polarization of the radiating source and its position with respect to the reflector both are important as one can control radiating properties ofthe overall antenna such as radiation pattern, directivity, impedance etc.
- Image theory has been used to analyze the radiation characteristics of such a antenna.

⇨ **Corner Reflector:**

- The disadvantage of the plane reflector is that there is radiation in back and side directions. Hence in order to overcome this limitation, the shape of the plane reflector is modified so that the radiation is in forward direction only.
- The modified arrangement consists of two plane reflector which are joined to form a corner with some angle. The reflector is thus known as corner reflector. The angle at which two plane reflectors are joined is called included angle (α). In most of the practical applications, the included angle is 90°.
- In some of the other applications angles other than 90° are also used. A typical corner reflector is shown in Fig. 2-17. The top view of the corner reflector is shown in Fig. 2-17 (a).
- The Fig. 2-17 (b) indicates the front view of the corner reflector. The vertical corner reflector with field pattern along main axis is shown in Fig. 2-17 (c).

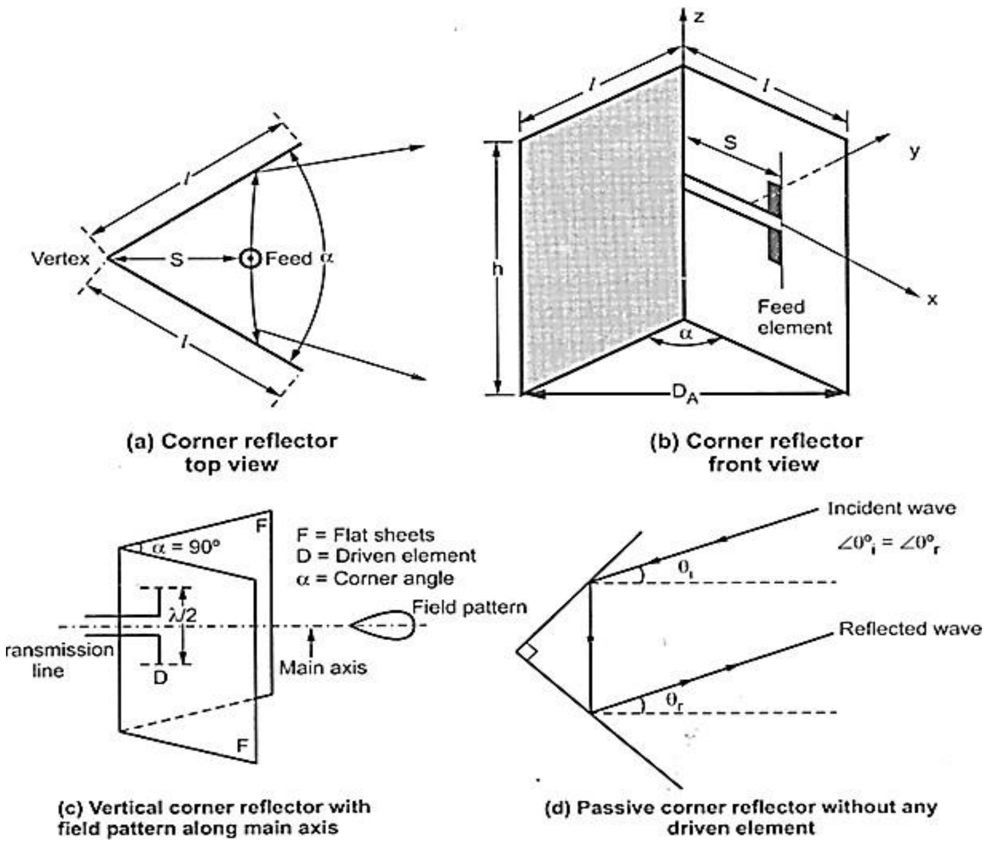


Fig. 2-17 Views of corner reflector antenna

- Thus when two flat reflecting sheets intersect each other at corner or at an angle, we get effective directional antenna called corner reflector. When the corner angle or included angle is 90°, the corner reflector is called square corner reflector.
- Practically corner reflectors with included angle less than 90° are not advantageous. When included angle tends to 180° we get a flat sheet reflector which can be considered as the limiting condition of corner reflector.
- The analysis of the corner reflector is carried out under the assumption that the two intersecting planes are perfectly conducting and infinite.
- In most of the corner reflectors, the feed element is either a dipole or array of collinear dipoles placed parallel to the vertex at a distance S as shown in Fig. 2-17 (b). To increases the bandwidth, instead of thin wires as feed element, the biconical or cylindrical dipoles are preferred.
- For the mathematical analysis, the dimensions specified are aperture of corner reflector (D_A) length of the reflector (l) and height (h). Generally the dimension of the aperture of the reflector (D_A) is selected between one and two wavelengths ($\lambda < D_A < 2\lambda$).
- The spacing between the vertex of the reflector and the feed element is selected as a fraction of wavelength ($\lambda/2 < S < 2\lambda$). The length of the reflectors is typically selected as twice the spacing between feed and vertex ($i.e.\ l \approx 2S$) for the included angle of 90°.
- The radiation resistance is the function of the spacing between the feed and the reflector. If the spacing is too large, the unwanted multiple lobes are produced and hence the directivity of the antenna is lost. If the spacing is very small, the radiation resistance decreases.
- The losses in the system increase as the decreased radiation resistance becomes comparable with the loss resistance of the antenna. Thus antenna is treated as inefficient antenna.
- The height of the reflector (h) is generally selected as about 1.2 to 1.5 times greater than the total length of the feed element.
- A corner reflector with two flat conducting sheets at a corner angle α and a driven antenna is called *active corner reflector antenna* or simply corner reflector antenna.
- If the corner reflector antenna consists only two flat conducting sheet at a corner angle α without any driven element then it is called *passive corner reflector antenna*. Such a passive corner reflector antenna is as shown in Fig. 2-17 (d).

⇨ **PARABOLIC REFLECTOR**

- To improve the overall radiation characteristics of the reflector antenna, the parabolic structure is oftenly used. Basically a parabola is a locus of a point which moves in such a way that the distance of the point from fixed point called focus plus the distance from the straight line called directrix is constant as shown in Fig. 2-18.
- By the definition ; $FM + MM' = FN + NN' = FP + PP' = constant$;
- By the geometrical optics (GO), when the point source is placed at the focus or focal point, then the rays reflected by the parabolic reflector form parallel wave front as shown in Fig. 2-18 (b). This principle is used in the transmitting antenna.

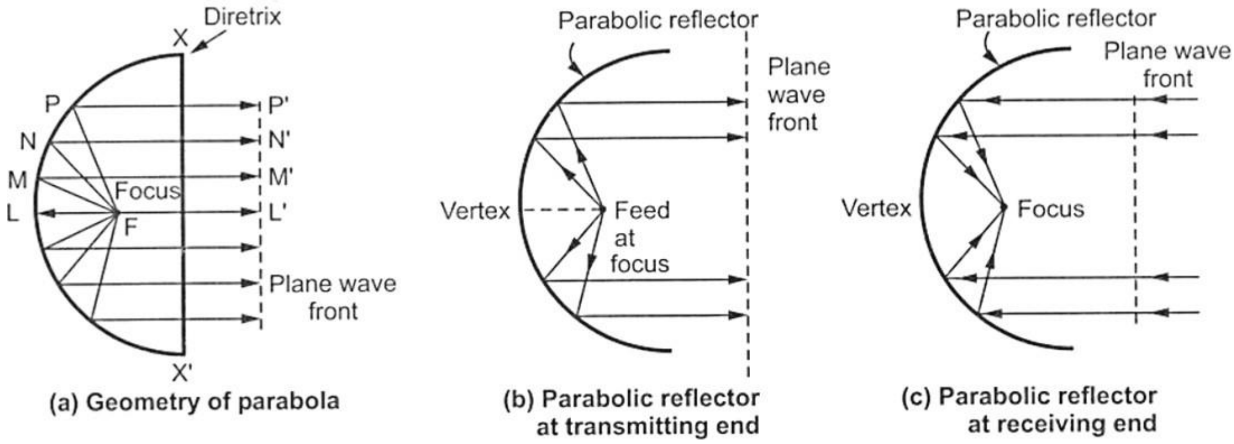


Fig. 2-18 Parabolic reflector principle

- Similarly when the beam of parallel rays is incident on a parabolic reflector, then the radiations focus at a focal point as shown in Fig. 2-18 (c). This principle is used in the receiving antenna.
- Consider a parabolic reflector as shown in Fig. 2-18 (b). When the point source is kept at the focus or focal point of the parabola, the radiations striking the parabolic reflector are reflected parallel to the axis of parabola irrespective of the angle at which the radiations strike the reflector. That means the rays which are reflected by the parabolic reflector travel same distance to reach near the mouth of the reflector.
- The open end of the parabolic reflector is called aperture. The time taken by the reflected rays to travel a distance upto the directrix of the parabola is same. That means all the reflected rays are in phase. Thus the wave front at the aperture of the parabolic reflector is uniform phase front and thus very strong and concentrated beam is obtained along the axis.
- Thus parabolic reflector is the most effective microwave antenna which produces concentrated radiation beam along the axis of parabola. The power gain of the paraboloid is a function of ratio between diameter of aperture.

⇨ **Paraboloid or Paraboloidal Reflector or Microwave Dish Antenna:**

- The parabolic reflector is a two dimensional structure. In practical applications, a three dimensional structure of the parabolic reflector is used. The three dimensional structure of the parabolic reflector can be obtained by rotating the parabola around its axis and it is called paraboloid.

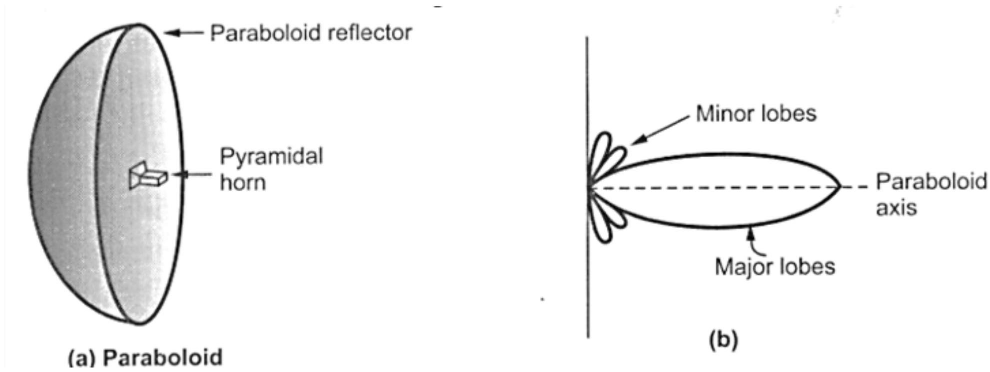


Fig. 2-19 Paraboloid with pyramidal horn as feed

- The paraboloid is as shown in Fig. 2-19 (a). The radiation pattern of the paraboloid is as shown in Fig. 2-19 (b). As the mouth of the paraboloid is circular in shape, the parallel, beam produced are of the circular cross-section. The radiation pattern consists very sharp major lobe and smaller minor lobes.
- Consider that the power gain of the paraboloid, with circular mouth or aperture, with respect to half wave dipole is given by,

$$G = \frac{4\pi A_0}{\lambda^2} \qquad \text{-------- (2.49)}$$

- Here A_0 is the capture area which is less than the actual area A_e of the mouth and it is given by,

$$A_0 = k.\, A_e \qquad \text{-------- (2.50)}$$

where $k =$ constant dependent on feed antenna used. It is 0.65 for dipole.

- The actual area of circular aperture with diameter d is given by

$$A_e = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} \qquad \text{-------- (2.51)}$$

- Hence, the power gain is given by;

$$G = \frac{4\pi(kA_e)}{\lambda^2} = \frac{4\pi \times 0.65\, A_e}{\lambda^2} = 6 \left(\frac{d}{\lambda}\right)^2 \qquad \text{-------- (2.52)}$$

- The above equation clearly indicates that the power gain of the paraboloid depends on the ratio or diameter d of the circular aperture to the wavelength in free space. The ratio d/λ is called aperture ratio of the paraboloid. Hence the effective radiated power (ERP) is the product of the input power fed and the power gain G.
- With small diameter of the paraboloid, the gain of the paraboloid is extremely large when λ is small in microwave frequency range. Consider $\lambda = 0.02\ m$ and diameter $d = 1\ m$, then the power gain of the paraboloid is given by ;

$$G = 6 \left(\frac{1}{0.02}\right)^2 = 15000 \qquad \text{-------- (2.53)}$$

- Hence if the input power fed to the paraboloid is 1 $watt$ at 1.5 GHz then the effective radiated power is 15 kW.
- For lower frequencies such as VHF, λ is large. Thus the diameter of the circular aperture becomes too large. Hence practically use of the parabolic reflectors are avoided at lower frequency.
- If the feed antenna is isotropic (called primary antenna), then paraboloid produces beam of radiation. Assuming large circular aperture, the FNBW is given by ;

$$FNBW = \frac{140\lambda}{d}\ degree \qquad \text{-------- (2.54)}$$

where ; $d =$ diameter of circular aperture in terms of λ in m.

- Similarly for uniformly illuminated rectangular aperture ;

$$FNBW = \frac{115\lambda}{d}\ degree \qquad \text{-------- (2.55)}$$

where ; $L =$ length of rectangular aperture in terms of λ in m.

- The HPBW for large circular aperture is given by ;

$$HPBW = \frac{58\lambda}{d}\ degree \qquad \text{-------- (2.56)}$$

- Similarly the directivity of uniformly illuminated aperture ;

$$D = \frac{4\pi A_e}{\lambda^2} = 9.87\left(\frac{d}{\lambda}\right)^2 \qquad [\because A_e = \frac{\pi d^2}{4}] \qquad \text{-------- (2.57)}$$

- Thus , from the above equations, it is clear that a sharp, concentrated pencil beam can be easily obtained at microwave frequencies using the paraboloid as illustrated in Fig. 2-20.

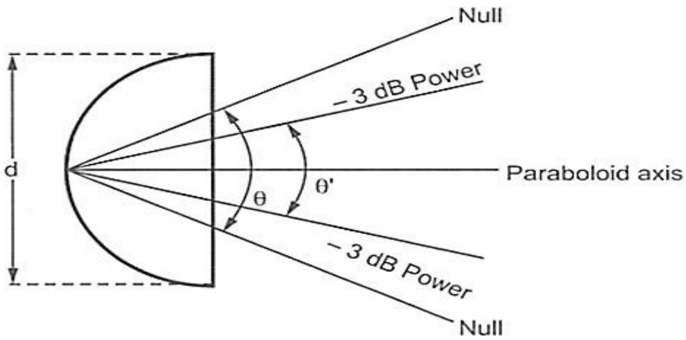
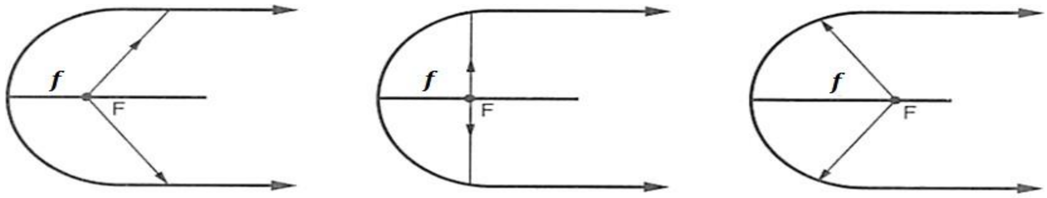


Fig. 2-20 Illustration of FNBW and HPBW

⇨ ***f*/ , SpillOver, BackLobe:**

- In paraboloid reflector, the ratio of the focal length f to the diameter of aperture is another important design constraint. The paraboloid can be designed to obtain pencil shape radiation beam by keeping the diameter of the aperture fixed and changing the focal length f.
- The three possible cases are as follows ;
    - Focal point inside the aperture ofparaboloid.
    - Focal point along the plane of open mouth of paraboloid.
    - Focalpoint beyond the open mouth of paraboloid
- When the focal length is very small, the focal point lies inside the open mouth of paraboloid as shown in Fig. 2-21 (a). It is very difficult to obtain uniform illumination over a wide angle. When the focal point lies on the plane of the open mouth of the paraboloid by the geometry, the focal length f is one fourth of the open mouth diameter d.
- This condition gives maximum gain pencil shaped radiation equal in horizontal and vertical plane. It is represented in Fig. 2-21 (b). Whenthe focal length is too large, the focal point lies beyond the open mouth of the paraboloid as shown in Fig. 2-21 (c). Here it is difficult to direct all the radiations from the source on the reflector.
- For practical applications, the value of the focal length to diameter ratio lies between 0.25 to 0.5.



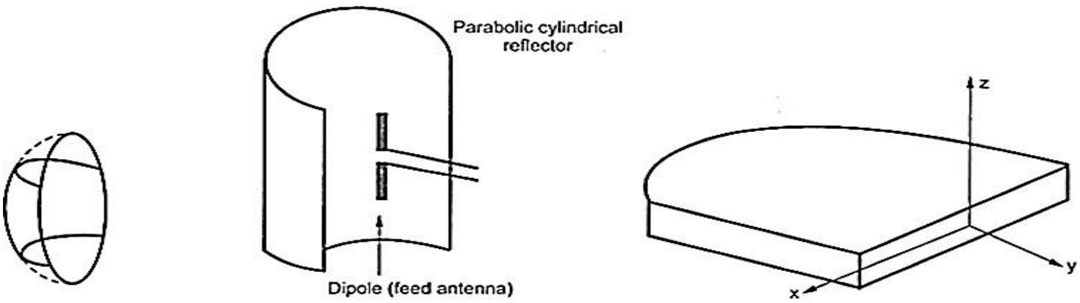
a) $f < \frac{d}{4}$ b) $f = \frac{d}{4}$ c) $f > \frac{d}{4}$

Fig. 2-21 Effect of variation of focal length f keeping diameter of aperture d fixed in paraboloid

- In addition to the desired radiation, some of the desired rays are not fully captured by reflector, such non-captured rays form *spill over*. While receiving spill over, the noise pick up increases which is troublesome.
- In addition to this, few radiations originated from the primary radiators are observed in forward direction such radiations get added with desired parallel beam. This is *called back lobe radiation* as it originates from the back lobe of primary radiator. Obviously the back lobe radiations are unwanted as they considerably affect the reflected beam.

⇨ **<u>Types of Paraboloidal Reflectors:</u>**

- Depending upon the use the paraboloid is modified in various types of the structures. Some of the important types of the paraboloid are as follows.
- **Truncated paraboloid or cut paraboloid:** This type of the paraboloid is formed by cutting some of the portion of the paraboloid to meet the requirements. As the portion of the paraboloid is cut away or truncated as shown in Fig. 2-22 (a) , the paraboloid is called cut paraboloid or truncated paraboloid.



(a) (b) (c)

Fig. 2-22 a) Truncated paraboloid or cut paraboloid b) Parabolic right cylinder c) Pill box or cheese antenna

- **Parabolic right cylinder:** The right cylindrical structure of the parabolic reflector is as shown in Fig. 2-22 (b). This structure is obtained by moving the parabola side ways. This parabolic structure has focal line instead of a focal point and similarly a vertex line instead of avertex. In parabolic right cylinder reflector the energy is collimated at a line which is parallel to the axis through the focal point of the reflector. In practice, linear dipole or linear array or a slotted waveguide is used as primary antenna.

- **Pill box or cheese antenna:** The cheese antenna or pill box is a short parabolic right cylinder enclosed by parallel plates as shown in Fig. 2-22 (c). This antenna is useful in producing wide beam in one of the planes while a narrow in other.

⇨ **Feed systems of Paraboloidal Reflectoror:**

- A parabolic reflector antenna system consists of two basic parts namely a source of radiation focus and a reflector. The source placed at the focus is called primary radiator, while the reflector is called secondary radiator. The primary radiator i.e. the source is commonly called feed radiator or simply feed.
- In case of a parabolic reflector a feed is said to be ideal feed, if it radiates entire energy towards the reflector in such a way that the entire surface of reflector is illuminated and no energy is radiated in any unwanted direction.
- Practically there are number of possible feeds to the parabolic reflector antenna. The secondary radiator used is most of the times a paraboloid.
- The simplest type of the feed that can be used is a dipole antenna. But it is not a suitable feed for the parabolic reflector antenna. Instead of only dipole, a feed consisting dipole with parasitic reflectors (like Yagi-Uda antenna) can be used as a feed system. In such cases, the spacing between the dipole as a driven element and the parasitic reflector is 0.125 . In some cases a dipole along with a plane reflector spaced 0.4 λ apart from the dipole is used. It is shown in Fig. 2-22 (a).
- In some cases, an end fire array of dipoles is used as feed radiator as shown in Fig. 2-22 (b). The dipoles are spaced in such a way that the end fire pattern of an array, illuminates reflector.
- The most widely used feed system in the parabolic reflector antenna is horn antenna as shown in Fig. 2-22 (c). The horn antenna is fed with a waveguide. In case, if circular polarization is required, then in place of a rectangular horn, a conical horn or helix antenna is used at the focus.
- In all cases, the feed or primary radiator is placed at the focus to obtain maximum beam pattern. If the feed is moved along a line perpendicular to the main axis then beam deteriorates. But if the feed is moved along the main axis, then the beam gets broadened. Hence focus is the important point on the main axis at which the feed is placed to obtain maximised beam pattern.

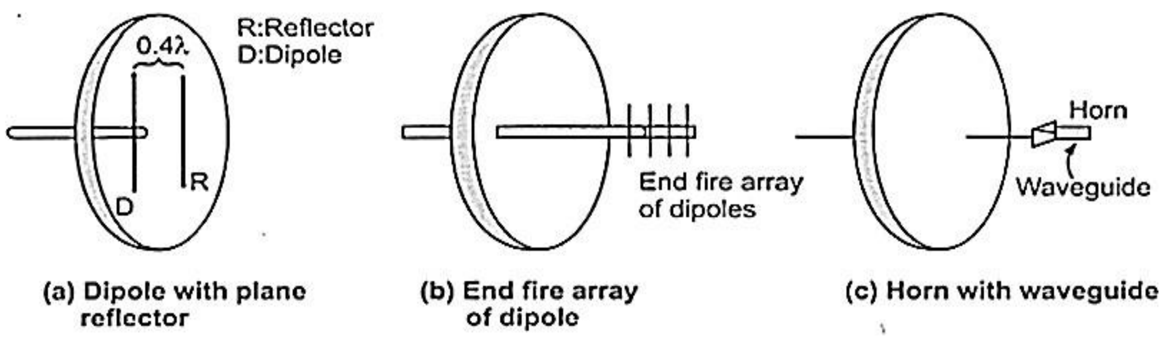


Fig. 2-22 Feed system

⇨ **Cassegrain Feed:**

- This system of feeding paraboloid reflector is named after a mathematician Prof. Cassegrain. In all the feed systems, the feed is located at the focus. But in Cassegrain feed system, the feed radiator is placed at the vertex of the parabolic reflector, instead of placing it at the focus.

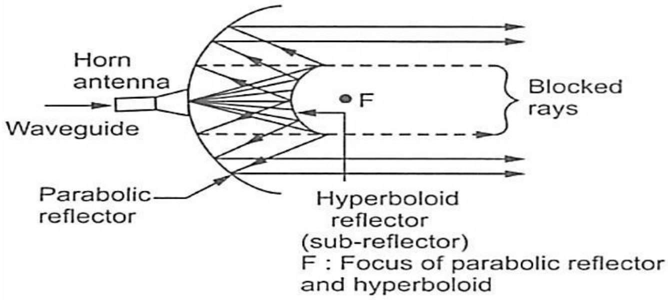


Fig. 2-24 Cassegrain feed system

- This system uses a hyperboloid reflector, such that its one of the foci coincides with the focus of the parabolic reflector. This hyperboloid reflector is called Cassegrain secondary reflector or sub-reflector. The primary radiator or feed radiator used is generally a horn antenna.
- The radiation emitted from primary feed radiator reach sub-reflector. The sub reflector reflects and illuminates the main parabolic reflector. The main reflector reflects the rays parallel to the axis. The geometry ofthe Cassegrain feed system is as shown in Fig. 2-24.

Advantages of Cassegrain Feed System

- It reduces the spill over and thus minor lobe radiations.
- With this system greater focal length greater than the physical focal length can be achieved.
- The system has ability to place a feed at convenient place.
- Using this system, beam can be broadened by adjusting one of the reflector surfaces.

Disadvantages of Cassegrain Feed System

- It is clear that there is a region of blocked rays infront of Cassegrain reflector. Some of the radiation from the parabolic reflector are obstructed or blockedby the hyperboloid reflector creating region of blocked rays. It is called ***aperture blockage***. This is not very serious problem in case of a parabolic reflector of larger dimensions.
- But for small dimension parabolic reflector it is the main drawback of the Cassegrain feed system.

⇨ **Offset FeedSystem**

- To overcome the aperture blocking effect due to the dependence of the secondary reflector dimensions on the distance between feed and sub-reflector, the offset feed system as shown in Fig. 2-25 is used. Here feed radiator is placed at the focus. With this system all the rays are properly collimated without formation of the region of blocked rays.

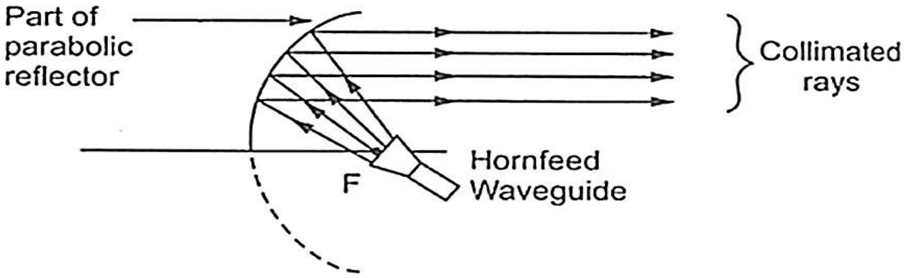


Fig. 2-25 Offset feed system

MICROSTRIP PATCH ANTENNA

- Microstrip patch antenna consists of a radiating patch on one side of a dielectric substrate which has a ground plane on the other side as shown in Fig. 2-26. The patch is generally made of conducting material such as copper or gold and can take any possible shape. The radiating patch and the feed lines are usually photo etched on the dielectric substrate.

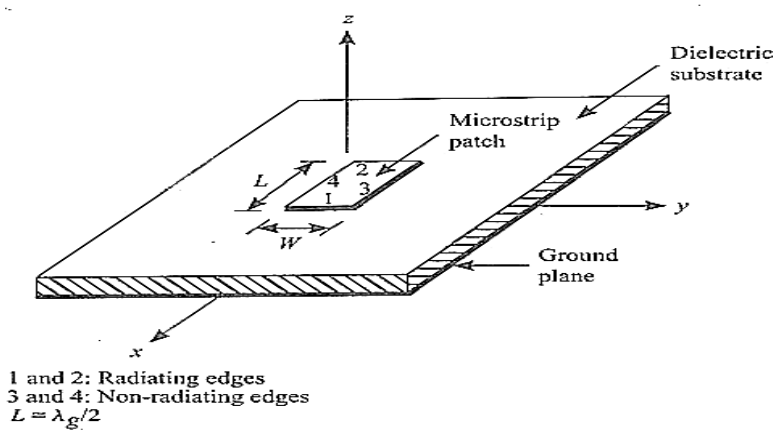


Fig. 2-26 Structure of a Microstrip Patch Antenna (MSA)

- In order to simplify analysis and performance prediction, the patch is generally square, rectangular, circular, triangular, elliptical or some other common shape as shown in Fig. 2-27.

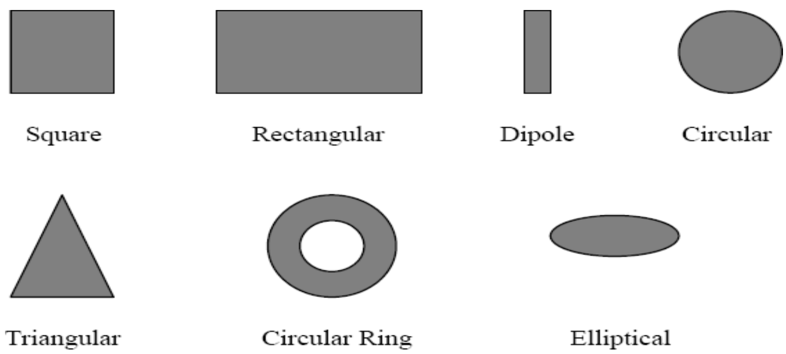


Fig. 2-27 Representative shapes of microstrip patch elements.

- For a rectangular patch, the length L of the patch is usually $0.333\lambda_0 \leq h \leq 0.5\lambda_0$, where λ_0 is the free-space wavelength. The patch is selected to be very thin such that $<< \lambda_0$ (where t is the patch thickness). The height h of the dielectric substrate is usually $0.033\lambda_0 \leq h \leq 0.05\lambda_0$. The dielectric constant of the substrate (ε_r) is typically in the range $2.2 \leq \varepsilon_r \leq 12$.
- Microstrip patch antennas radiate primarily because of the fringing fields between the patch edge and the ground plane. For good antenna performance, a thick dielectric substrate having a low dielectric constant is desirable since this provides better efficiency, larger bandwidth and better radiation.
- However, such a configuration leads to a larger antenna size. In order to design a compact microstrip patch antenna, higher dielectric constants must be used which are less efficient and result in narrower bandwidth. Hence a compromise must be reached between antenna dimensions and antenna performance.

⇨ **Mechanism of radiation:**

- Consider a rectangular patch of length L and width W printed on a dielectric substrate of height h. The length of the patch is chosen to be around $\lambda_g/2$, where λ_g is the guide wavelength of the microstrip line of width W printed on the same dielectric substrate.
- With this choice of the length, the electric field along x-direction undergoes a 180° phase reversal (Fig. 2-28) from one edge to the other. It can be shown that the fields near edges 1 and 2 constructively add up producing the radiation with a maximum along the z-direction.

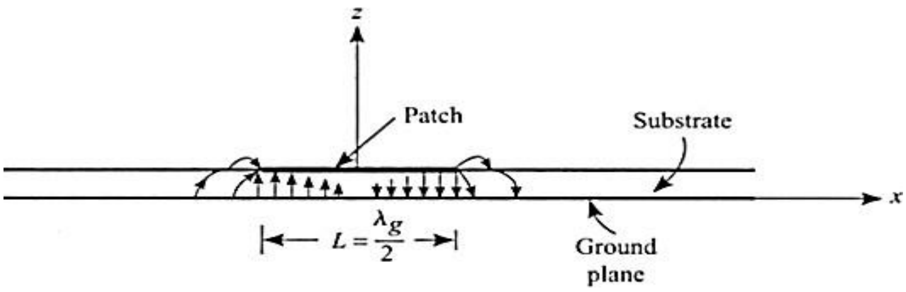


Fig. 2-28 Electric field distribution in a microstrip patch antenna

- Hence, edges 1 and 2 are known as the radiating edges. Further, it has been shown that the fields near edges 3 and 4 do not contribute to the radiation. The rectangular microstrip patch shown in Fig. 4-24 radiates linearly polarized waves, with the electric field oriented along the x-direction when looking in the direction of maximum radiation.
- The radiation patterns in the two principal planes, viz., the E-plane (x-z plane) and the H-plane (y-z plane) are shown in Fig. 2-29. The pattern is very broad and has nulls along the y-direction.

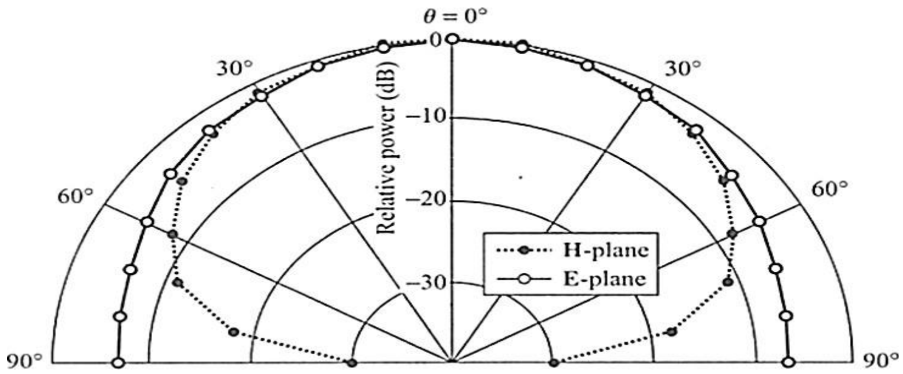


Fig. 2-29 Radiation patterns ofa microstrip patch antenna

- For efficient transfer of power from a transmission line to the patch antenna, we need to match the input impedance of the antenna to the characteristic impedance of the transmission line.
- It is observed that the impedance seen by a transmission line attached to the radiating edge is very high, and also the impedance (ratio of voltage to current) decreases as one moves towards the centre of the patch. Therefore, depending on the characteristic impedance of the transmission line, an appropriate point on the patch is chosen as the feed point.

⇨ **Feeding Techniques**

- Microstrip patch antennas can be fed by a variety of methods. The four most popular feed techniques used are the microstrip line, coaxial probe, aperture coupling and proximity coupling.

Microstrip Line Feed

- In this type of feed technique, a conducting strip is connected directly to the edge of the microstrip patch as shown in Fig. 2-30. The conducting strip is smaller in width as compared to the patch. This kind of feed arrangement has the advantage that the feed can be etched on the same substrate to provide a planar structure.
- An inset cut can be incorporated into the patch in order to obtain good impedance matching without the need for any additional matching element. This is achieved by properly controlling the inset position. Hence this is an easy feeding technique, since it provides ease of fabrication and simplicity in modeling as well as impedance matching.
- However as the thickness of the dielectric substrate increases, surface waves and spurious feed radiation also increases, which hampers the bandwidth of the antenna. This type of feeding technique results in undesirable cross polarization effects.

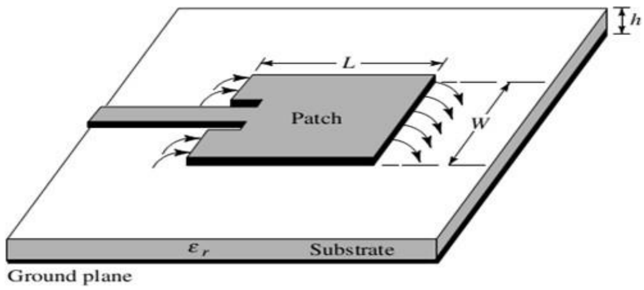


Fig. 2-30 Microstrip Line Feed

Coaxial Feed

- The Coaxial feed or probe feed is one of the most common techniques used for feeding microstrip patch antennas. As seen from Fig. 2-31, the inner conductor of the coaxial connector extends through the dielectric and is soldered to the radiating patch, while the outer conductor is connected to the ground plane.

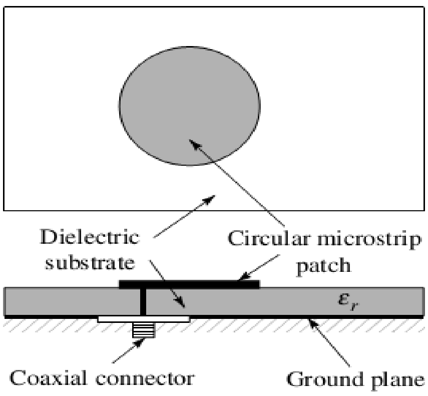


Fig. 2-31 Coaxial feed

- The main advantage of this type of feeding scheme is that the feed can be placed at any desired position inside the patch in order to obtain impedance matching. This feed method is easy to fabricate and has low spurious radiation effects. However, its major disadvantage is that it provides narrow bandwidth and is difficult to model since a hole has to be drilled into the substrate. Also, for thicker substrates, the increased probe length makes the input impedance more inductive, leading to matching problems.
- By using a thick dielectric substrate to improve the bandwidth, the microstrip line feed and the coaxial feed suffer from numerous disadvantages such as spurious feed radiation and matching problem. The non-contacting feed techniques which have been discussed below, solve these problems.

<u>Aperture Coupled Feed</u>

- In aperture coupling the radiating microstrip patch element is etched on the top of the antenna substrate, and the microstrip feed line is etched on the bottom of the feed substrate in order to obtain aperture coupling. The thickness and dielectric constants of these two substrates may thus be chosen independently to optimize the distinct electrical functions of radiation and circuitry.
- The coupling aperture is usually centered under the patch, leading to lower cross polarization due to symmetry of the configuration. The amount of coupling from the feed line to the patch is determined by the shape, size and location of the aperture. Since the ground plane separates the patch and the feed line, spurious radiation is minimized as shown in Fig. 2-32.

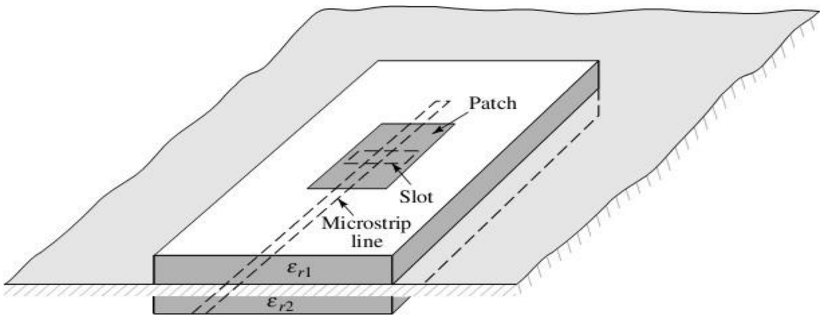


Fig. 2-32 Aperture coupled feed

- Generally, a high dielectric material is used for bottom substrate and a thick, low dielectric constant material is used for the top substrate to optimize radiation from the patch. This type of feeding technique can give very high bandwidth of about 21%. Also the effect of spurious radiation is very less as compared to other feed techniques. The major disadvantage of this feed technique is that it is difficult to fabricate due to multiple layers, which also increases the antenna thickness.

<u>Proximity Coupled Feed</u>

- This type of feed technique is also called as the electromagnetic coupling scheme. As shown in Fig. 2-33, two dielectric substrates are used such that the feed line is between the two substrates and the radiating patch is on top of the upper substrate.

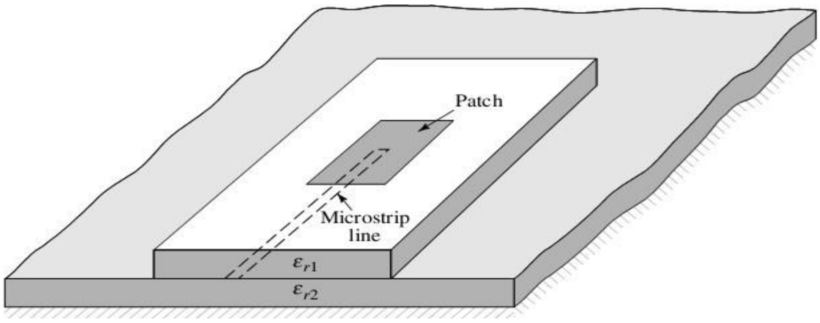


Fig. 2-33 Proximity coupled feed

- The main advantage of this feed technique is that it eliminates spurious feed radiation and provides very high bandwidth of about 13%, due to increase in the electrical thickness of the microstrip patch antenna. This scheme also provides choices between two different dielectric media, one for the patch and one for the feed line to optimize the individual performances.
- The major disadvantage of this feed scheme is that it is difficult to fabricate because of the two dielectric layers that need proper alignment. Also, there is an increase in the overall thickness of the antenna.

Comparison:

Characteristics	Microstrip Line Feed	Coaxial Feed	Aperture coupled Feed	Proximity coupled Feed
Spurious feed radiation	More	More	Less	Minimum
Reliability	Better	Poor due to soldering	Good	Good
Ease of fabrication	Easy	Soldering and drilling needed	Alignment required	Alignment required
Impedance Matching	Easy	Easy	Easy	Easy
Bandwidth (achieved with impedance matching)	2-5%	2-5%	2-5%	13%

⇨ **Advantages and Disadvantages:**

- Some of their principal advantages are ;
  - Light weight and low volume.
  - Low profile planar configuration which can be easily made conformal to host surface.
  - Low fabrication cost, hence can be manufactured in large quantities.
  - Supports both, linear as well as circular polarization.
  - Can be easily integrated with microwave integrated circuits (MICs).
  - Capable of dual and triple frequency operations.
  - Mechanically robust when mounted on rigid surfaces.

- Microstrip patch antennas suffer from a number of disadvantages as compared to conventional antennas. Some of their major disadvantages are
    - Narrow bandwidth
    - Low efficiency
    - Low gain
    - Extraneous radiation from feeds and junctions
    - Poor end fire radiator except tapered slot antennas
    - Low power handling capacity.

⇨ **Applications:**

- In high-performance aircraft, spacecraft, satellite, and missile applications, where size, weight, cost, performance, ease of installation, and aerodynamic profile are constraints,low-profile antennas may be required. To meet these requirements, microstrip antennas can be used.
- Other areas where microstrip antennas widely used are: GPS, Telemetry, Radars, Altimeters, etc.,

NUMERICAL TOOL FOR ANTENNA ANALYSIS

- The following are the list of numerical tools used for antenna analysis:

Software Name	Theoretical Model	Company
Ensemble (Designer)	Moment method	Ansoft
IE3D	Moment method	Zeland
Momentum	Moment method	HP
EM	Moment method	Sonnet
PiCasso	Moment method/genetic	EMAG
FEKO	Moment method	EMSS
PCAAD	Cavity model	Antenna Design Associates, Inc
Micropatch	Segmentation	Microstrip Designs, Inc.
Microwave Studio (MAFIA)	FDTD	CST
Fidelity	FDTD	Zeland
HFSS	Finite element	Ansoft

Post – MCQ:

1. Aperture antennas are most common at _____________

a.LHF

b.VHF

c.Microwave Frequencies

d.HF

Ans: c

2. Slot antenna is the best suitable radiator at frequencies above _______

a. Above 200MHz

b. Below 200MHz

c. Above 100MHZ

d. Below 100MHz

Ans: a

3. The radiations from the backside of the slot antenna and the complementary antenna are of _________ polarity

a.Same

b.Opposite

c.Negative

d.Positive

Ans:b

4. The horn is widely used as a ___________ for large radio astronomy, communication dishes.

a.Feed Element

b.Antenna Array

c.Transmission line

d.Matching Element

Ans:a

5. The Horn Antenna used as a feed element in antennas such as _______________

a.Parabolic Reflectors

b.Antenna Array

c.Log periodic Antenna

d.Slot antenna

Ans: a

6. Reflector antenna is used as ______________

a.Primary Antenna

b.Horn Antenna

c.Secondary Antenna

d.Dipole Antenna

Ans:c

7. Cassegrain Feed System reduces the ____________ and thus minor lobe radiations

a.More Radiation

b.Major Lobes

c.Side Lobes

d.Spill Over

Ans: d

8. The ______________ is one of the most common techniques used for feeding microstrip patch antennas

a.Coaxial Cable or Probe Feed
b.Slot Feed
c.Dipole Feed
d.Horn Feed

Ans: a

9. Microstrip Antennas are easily integrated with ___________

a.Microwave integrated circuits (MICs).
b.Integrated Circuits
c.Very Large Integrated Circuits
d.PCB Board

Ans: a

10. The Following is one of the Numerical Tool for Antenna Analysis __________

a.HFSS
b.MatLab
c.DSP Simulator
d.VisSim

Ans: a

11. Aperture antennas are very practical applications in ___________

a.spacecraft or aircraft
b.Automotive
c.Instrumentation
d.Agricultural

Ans: a

12. The infinite conducting sheet with slot and the flat strip of the dimension same as of the slot are said to be __________ antenna

a.Horn
b.Complementary
c.Plain Antenna
d.Cornor Antenna

Ans: b

13. The shape of the slot may be either

a.rectangular or circular
b.Elliptical
c.Triangular
d.Squire

Ans: a

14. The angle at which two plane reflectors are joined is called _______

a.Apex Angle
b. Included Angle
c. Phase Angle
d. Acute Angle

Ans: b

15. When the corner angle is 90°, the corner reflector is called

a.Cornor reflector
b.square corner reflector.
c.proximity coupling.
d.All the above

Ans: b

Conclusion:

At the end of the topic, students will be able –

1) To understand the basic concept of Aperture Antennas and their radiation characteristics.

2) To understand the different feeding structure of aperture antennas

3) To know the applications of Aperture antennas and different tools for antenna analysis.

References:

1. Constantine.A. Balanis “Antenna Theory Analysis and Design”, Wiley Student Edition, 4th Edition 2016.

2. Rajeswari Chatterjee, “Antenna Theory and Practice” Revised Second Edition New Age International Publishers, 2006.
3. S. Drabowitch, “Modern Antennas” Second Edition, Springer Publications, 2007
4. Robert S. Elliott “Antenna Theory and Design” Wiley Student Edition, 2006.

Assignments:

1. Sketch the various types of Horn Antenna and explain its operation.
2. Describe the principle of operation and applications of parabolic reflectors and derive its necessary equations.
3. Describe Flat sheet and corner reflectors and derive their field equations.
4. Explain the structure and operation of Slot antenna. Also derive the expression of its input impedance.
5. Explain in detail the radiation mechanism of a micro-strip patch antenna with diagram.

Subject Name: Antennas & Propagation

Topic Name: **Special Antennas**

(Unit – 4)

Syllabus / **Special Antennas**

1. Principle of frequency independent antennas –Spiral antenna, helical antenna, Log periodic.
2. Modern antennas- Reconfigurable antenna Active antenna, Dielectric antennas, Electronic band gap structure and applications
3. Antenna Measurements-Test Ranges, Measurement of Gain, Radiation pattern, Polarization, VSWR

Aim and Objective:

•To give insight basic Knowledge of Special Antennas

•To give thorough understanding of the radiation characteristics Special Antennas i.e Spiral antenna, helical antenna, Log periodic.

•To understand the concepts of Different Antenna Measurements and Modern Antennas.

Pre – Test MCQ:

1.Frequency independent antennas are used in _____________ region
a.$\mathbf{100 - 1000}\ \boldsymbol{MHz}$
b.$\mathbf{10 - 10,000}\ \boldsymbol{MHz}$
c.$\mathbf{1 - 10,000}\ \boldsymbol{MHz}$
d.None of the above
Ans: b

2.Frequency-independent antennas are governed by _________
a.$\boldsymbol{Rumsey's\ principle}$
b.Friss $\boldsymbol{ciple}$
c.Jordon $\boldsymbol{ciple}$
d.George $\boldsymbol{ciple}$
Ans: a

3.In log periodic array the Inactive stop region is
a.$\boldsymbol{l} \leq \lambda/2$
b.$\boldsymbol{l} > \lambda/2$
c.$\boldsymbol{l} \infty \lambda/2$
d.$\boldsymbol{l} \geq \lambda/2$
Ans: b

4.Helical antenna is a broadband VHF and UHF antenna, which is used to provide _________ polarization
a.Circular
b.Elliptical
c.Vertical
d.All the above
Ans: a

5.The helical antenna can operate in _______ different modes
a.normal (broadside) modes
b.the axial (end-fire) modes
c.None of the above
d.All the above
Ans: d

6.A reconfigurable antenna is an antenna capable of modifying dynamically ___________ properties in a controlled and reversible manner
a.its frequency and radiation
b.its Phase and Angle
c.It radiation and apex angle
d.Noneof the above
Ans: a

7. Active antenna components may consist of _________
a.amplifiers,
b.low-noise amplifiers (LNAs)
c.power amplifiers
d.All the above
Ans: d

8.slot antenna with a Schottky diode, which can be used for ________
a.frequency doubling (Ans)
b.Amplitude Doubling
c.Phase doubling
d.All the above
Ans: a

9.The dielectric antenna was first proposed in ________
a.1959.
b.1949.
c.1939.
d.1929.
Ans: c

10.The EBG applications are _______
a.GPS and low loss- coplanar lines
b. Bluetooth.
c.mobile telephony and waveguides.
d.All the above
Ans: d

Pre-requisite:

•Basic knowledge of Electromagnetic Fields and Wave guides.

•Basic Knowledge of Special Antennas and Antenna parameters.

UNIT IV SPECIAL ANTENNAS

Principle of frequency independent antennas – Spiral antenna, Helical antenna, Log periodic. Modern antennas- Reconfigurable antenna, Active antenna, Dielectric antennas, Electronic band gap structure and applications, Antenna Measurements-Test Ranges, Measurement of Gain, Radiation pattern, Polarization, VSWR

INTRODUCTION – FREQUENCY INDEPENDENT ANTENNA:

- The numerous applications of the electromagnetics need utilization of most of the electromagnetic spectrum. The invention of various broadband systems need the design of the broadband antennas.
- The antennas which are simple, small, light weight, economical and importantly operating over the entire frequency band are most desirable. Such antennas refered as frequency independent antenass are used in $\mathbf{10-10,000}\ \boldsymbol{MHz}$ region for practical applications such as TV, point to point communication, feeds for reflectors and lenses.
- According to the antenna scale model measurements if the shape of the antenna is specified completely by angles, then its performance would be independent of frequency.
- To have such practical infinite structures, the current on the structure should decrease with distance away from the input terminals. After a certain point, the current becomes negligible and then the structure beyond that point to infinity can be removed.
- Such truncated antenna practically has lower cut-off frequency and beyond this cutoff frequency the radiation characteristics of the truncated antenna and infinite structure are identical. The lower cut-off frequency is that for which the current at the point of truncation becomes negligibly small.
- The biconical antenna can be compltely specified by angles but the current along the structure does not reduce with distance away from the input terminals. Also its pattern does not have a form limiting with frequency.
- Rumsey proposed a general shape equation which has frequency independent impedance, pattern and polarization characteristics and with this shape the current distribution reduces to zero rapidly.

PRINCIPLE OF FREQUENCY INDEPENDENT ANTENNA - SPIRAL ANTENNA

- Spiral is a geometrical shape found in nature. A spiral can be geometrically described using polar coordinates. Let (r, θ) be a point in the polar coordinate system. The equation

$$r = r_0 e^{a\theta} \qquad \text{-------- (4.1)}$$

where, r_0 and a are positive constants, describes a curve known as a logarithmic spiral or an equiangular spiral. Taking natural logarithm on both sides of Eqn. (4.1) ;

$$\ln r = \ln r_0 + a\theta \qquad \text{-------- (4.2)}$$

Differentiating with respect to ;

$$\frac{1}{r}\frac{dr}{d\theta} = a \qquad \text{-------- (4.3)}$$

- Spiral is a geometrical shape found in nature. A spiral can be geometrically described using polar coordinates. Let (r, θ) be a point in the polar coordinate system. The equation

$$r = r_0 e^{a\theta} \qquad \text{-------- (4.4)}$$

where, r_0 and a are positive constants, describes a curve known as a logarithmic spiral or an equiangular spiral. Taking natural logarithm on both sides of Eqn. (4.4) ;

$$\ln r = \ln r_0 + a\theta \qquad \text{-------- (4.5)}$$

Differentiating with respect to ;

$$\frac{1}{r}\frac{dr}{d\theta} = a \qquad \text{-------- (4.6)}$$

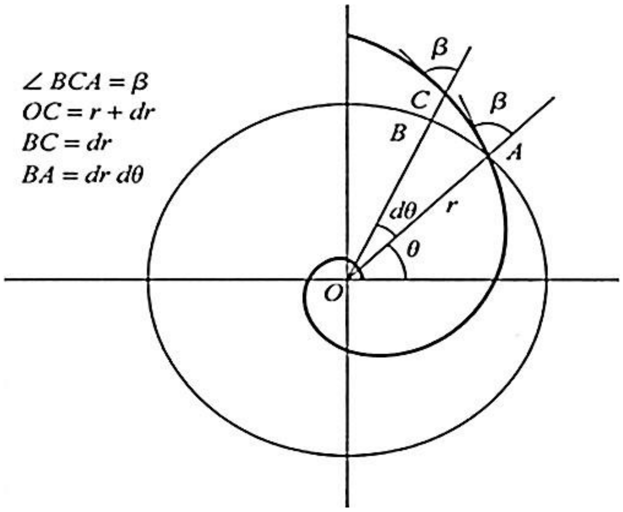


Fig. 4-1 Logarithmic spiral

- From Δ ABC in Fig. 4-1 ;

$$tan\beta = \frac{BA}{BC} = \frac{rd\theta}{dr} = \frac{1}{a} \qquad \text{-------- (4.7)}$$

- Therefore, the angle between the tangent at any point on the spiral and the radial line from the origin to that point (designated as β) is the same for all points on the spiral (a is a constant). Hence, the spiral represented by Eqn. (4.1) is also known as an equi-angular spiral.
- Consider a spiral described by

$$r_1 = r_0 e^{a\theta} \qquad \text{-------- (4.8)}$$

- The dimensions of an antenna designed to operate at a frequency, f_0. If the antenna is scaled by a factor K, it would have the same radiation and input properties at a frequency f_0/K.

  Multiplying Eqn. (4.8) by a factor K we have ;

$$r_2 = K r_0 e^{a\theta} \qquad \text{-------- (4.9)}$$

- Expressing $K = e^{a\delta}$, Eqn. (4.8) reduces to ;

$$r_2 = e^{a\delta} r_0 e^{a\theta} = r_0 e^{a(\theta+\delta)} \qquad \text{-------- (4.10)}$$

- This shows that the scaled antenna is obtained by rotating the original antenna structure by an angle δ. The structure itself is unchanged. Hence, the radiation pattern alone rotates by an angle δ, keeping all the other properties the same. Such an antenna is known as a *frequency-independent antenna.*

- Frequency-independent antennas are governed by ***Rumsey's principle***, which states that the impedance and pattern properties of an antenna will be frequency independent if the antenna shape is specified only in terms of angles. The antenna described by Eqn. (4.9) satisfies this criterion provided the structure is infinite.
- For structures that are finite in size, the frequency invariance property is exhibited over a limited range of frequencies. The lower end of this band is decided by the largest dimension of the spiral and the upper end by the smallest dimension.

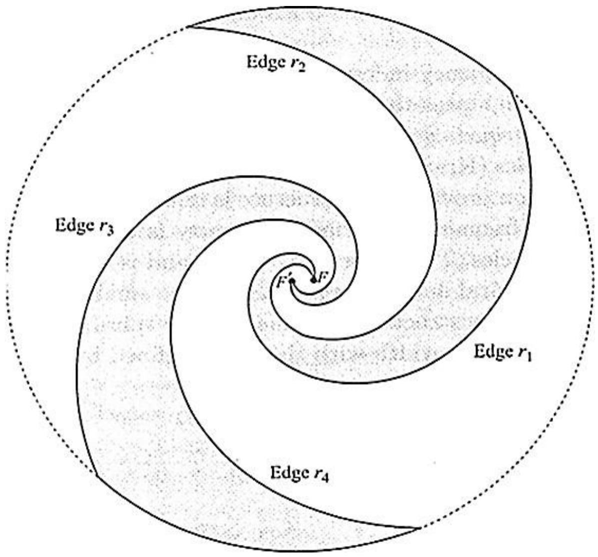


Fig. 4-2 An antenna based on a logarithmic spiral

- To construct an antenna using a spiral, consider a thin conducting strip of variable width with the edges defined by the following two equations ;

$$r_1 = r_0 e^{a\theta} \quad \text{-------- (4.10)}$$

and

$$r_2 = r_0 e^{a(\theta-\delta)} \quad \text{-------- (4.11)}$$

- These two edges are shown in Fig. 4-2 for $0 \geq \theta \leq 2.25\pi$ *and* $\delta \geq \theta \leq (2.25\pi + \delta)$. A second conductor can be obtained by rotating the first spiral by 180°. The edges of the second spiral are given by ;

$$r_3 = r_0 e^{a(\theta+\pi)} \quad \text{-------- (4.12)}$$

and

$$r_4 = r_0 e^{a(\theta+\pi-\delta)} \quad \text{-------- (4.13)}$$

- These edges, edge r_3 and edge r_4 are shown in Fig. 4-2 for $-\pi \geq \theta \leq 1.25\pi$ and $(-\pi + \delta) \geq \theta \leq (1.25\pi + \delta)$. These two conductors form a balanced structure with feed points *FF'.*
- The parameters used to define this structure are ;

  δ : determines the width of the arm

  r_0 : determines the radius of the feed region

a : rate of growth of the spiral, and

θ_{max} : determines the maximum radius of the spiral.

- The spiral antenna has a bidirectional main lobe perpendicular to the plane of the antenna. The radiated field is right circularly polarized on one side and is left circularly polarized on the other side of the spiral. The axial ratio is used as one of the convenient parameters to define the acceptable bandwidth of the antenna. Outside the band of operation of the antenna, the radiation is elliptically polarized.

LOG PERIODIC ANTENNA

- In general, any antenna when defined in terms of angles, then it comes under the category of the frequency independent antenna. Its charcteristics are found to be frequency independent. In any frequency independent antenna, the impedance and the radiation pattern both are independent of frequency.
- In order to be frequency independent, the antenna should expand or contract in proportion to the wavelength. If the antenna structure is not mechanically adjustable, the size of the radiating region should be proportional to wavelength.
- The log periodic antenna is a broadband antenna in which the geometry of the antenna structure is adjusted such that all the electrical properties of the antenna are repeated periodically with the logarithm of the frequency. Thus, the basic geometric structure is repeated with the structure size changed.
- For every repetition, the structure size changes by a constant scale factor, with which the structure can either expand or contract. Thus the principle of the log periodic antenna can be understood with the help of a array of the log periodic antenna known as ***Log Periodic Dipole Array*** **(LPDA)**.

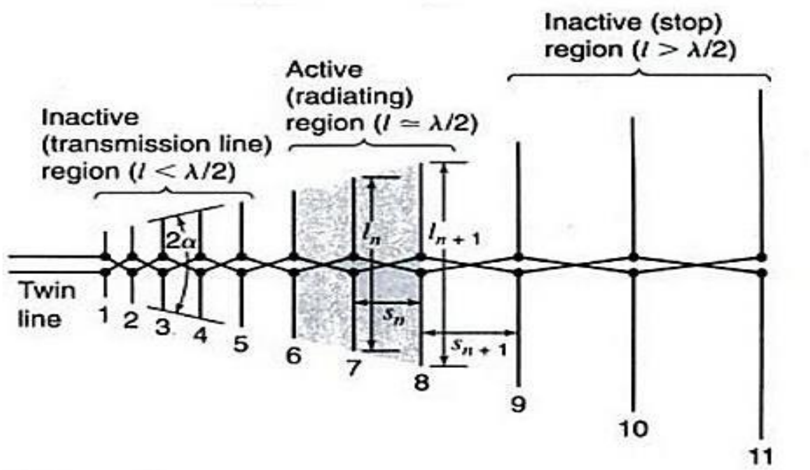


Fig. 4-3 Log periodic dipole array

- A typical log periodic dipole array (LPDA) consists number of dipoles of different lengths and spacings and is fed by balanced two wire transmission line as shown in Fig. 4-3. The feed line is connected at narrow end or apex of the array. The length of the dipoles increases from feed point towards other end such that the included angle α remains constant.

- The increase in the length of the dipole (l) and the spacing in wavelength between two dipoles (s) are adjusted such that the dimensions of the adjacent dipoles posses certain ratio with each other.
- The dipole lengths and the spacings between two adjacent dipoles are related through parameter called design ratio or scale factor which is denoted by τ. Thus the relationship between s_n and s_{n+1} and l_n and l_{n+1} is given by ;

$$\frac{l_{n+1}}{l_n} = \frac{s_{n+1}}{s_n} = k = \frac{1}{\tau} \qquad \text{-------- (4.14)}$$

- The ends of the dipoles lie along straight lines on both the sides. These two straight line meet at fixed point or apex giving angle 2α which is angle included by two straight line. Depending on the length of the diploles, there are three regions in LPDA, namely inactive transmission line region, active region and inactive stop region.
- **Inactive transmission line region** ($l < \lambda/2$)**:** It is the region in which the length of the dipole is less than $\lambda/2$. The elements in this region provide capacitive impedance. The element spacing in this region is comparatively smaller. The currents in the region are very small hence it is considered as inactive region. These currents lead the voltage supplied by the transmission line.
- **Active region** ($l \approx \lambda/2$)**:** In this region, the length of the dipoles are approximately equal to $\lambda/2$ i.e., equal to resonant length. This is the central region of the array from where maximum radiation takes place. In this region, the dipoles offer resistive impedance. Thus the currents are large value and in phase with the base voltage.
- **Inactive stop region** ($l > \lambda/2$)**:** In this region, the length of the dipoles are greater than $\lambda/2$ i.e., greater than resonant length. The dipoles offer inductive impedance. The currents are smaller in this region and also lags the base voltage. This is also called reflective region as small incident wave gets reflected due to the large inductive impedance.
- Thus for a wavelength λ, the radiation occurs from active region which is in the middle of the array. When the wavelength increases, the radiation zone moves towards right side of the active region while the wavelength decreases, the radiation zone moves towards left side of the active region.
- To find the relationship between the apex angle α, spacing s, and length l , consider part of the LPDA as shown in Fig. 4-4.

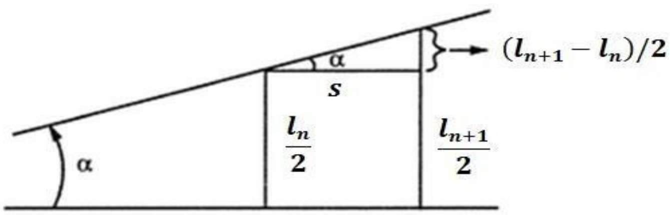


Fig. 4-4 Geometry of LPDA

- From Fig. 4-4 ;

$$\tan\alpha = \frac{(l_{n+1} - l_n)/2}{s} = \frac{l_{n+1} - l_n}{2s} \qquad \text{-------- (4.15)}$$

$$\tan\alpha = [l_{n+1}(1 - l_n/l_{n+1})]/2s$$

But ; $$\frac{l_{n+1}}{l_n} = k = \frac{1}{\tau}$$

$$\tan\alpha = \frac{[1-(1/k)](l_{n+1}/2)}{s} \qquad \text{-------- (4.16)}$$

For active region: $l_{n+1} = \lambda/2$

$$\tan\alpha = \frac{[1-(1/k)]}{4(s/\lambda)}$$

$$\tan\alpha = \frac{[1-(1/k)]}{4s_\lambda} \qquad \text{-------- (4.17)}$$

where α = apex angle, k = scale factor , s_λ =spacing in wavelength shortward of $\lambda/2$ element.

- The length of any element say $n+1^{th}$ element and length of first element is related as ;

$$\frac{l_{n+1}}{l_1} = k^n = F \qquad \text{-------- (4.18)}$$

- When the length of the first element is l_1 then the length $n+1^{th}$ element is k^n time greater than l_1. This ratio is also termed as frequency ratio F or it is called Bandwidth. The relation between the apex angle α, scale factor k and spacing s_λ with optimum design line and gain represented in Fig. 4-5.

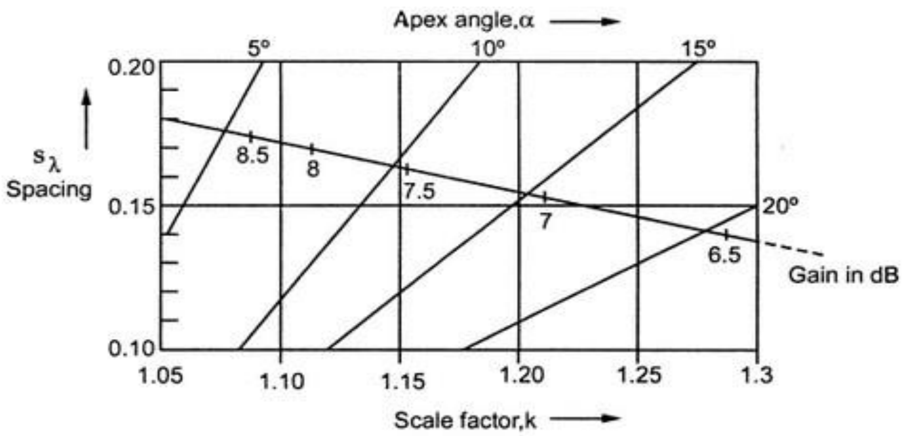


Fig. 4-5 Design curve of LPDA

- The number of elements in the array (n) can be obtained from upper frequency f_U and lower frequency f_L as ;

$$\log(f_U) - \log(f_L) = (n-1)\log\left(\frac{1}{\tau}\right) \qquad \text{-------- (4.19)}$$

HELICAL ANTENNA:

- Helical antenna is a broadband VHF and UHF antenna, which is used to provide circular polarization. It consists of a thick copper wire wound in the form of a screw thread forming a helix.

- In general , helix is used with a ground plane. There are different forms of ground plane such as flat ground plane, cylindrical cavity. The helix is usually connected to the center conductor of a coaxial transmission line at the feed point with the outer conductor of the line attached to the ground plane is as shown in Fig. 4-6.

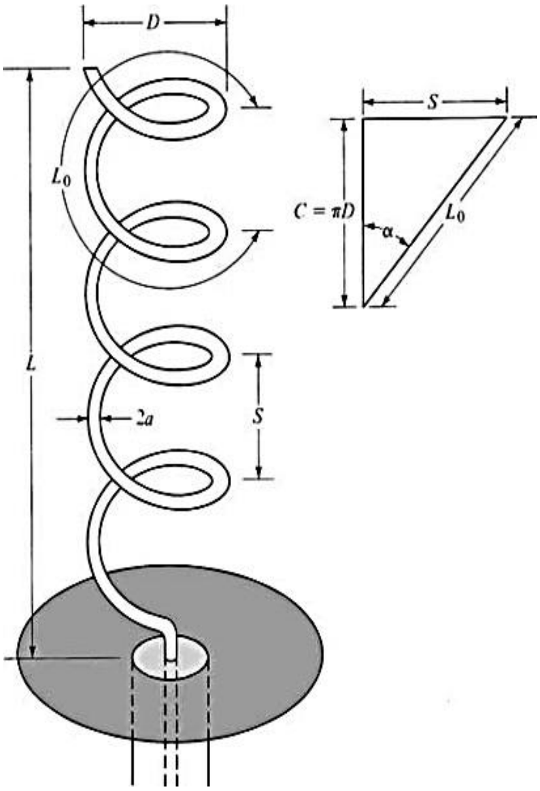


Fig. 4-6 Helical antenna with ground plane

- The geometrical configuration of a helix consists usually of N turns, diameter D and spacing S between each turn. The total length of the antenna is $L = NS$ while the total length of the wire is $L_n = NL_0 = N\sqrt{S^2 + C^2}$, where $L_0 = \sqrt{S^2 + C^2}$ is the length of the wire between each turn and $C = \pi D$ is the circumference of the helix.

- Another important parameter is the pitch angle α which is the angle formed by a line tangent to the helix wire and a plane perpendicular to the helix axis. The pitch angle is defined by ;

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right) = \tan^{-1}\left(\frac{S}{C}\right)$$

- When $\alpha = 0°$, then the winding is flattened and the helix reduces to a loop antenna of N turns. On the other hand, when $\alpha = 90°$ then the helix reduces to a linear wire. When $0° < \alpha < 90°$, then a true helix is formed with a circumference greater than zero but less than the circumference when the helix is reduced to a loop ($\alpha = 0°$).

Modes of operation:

The helical antenna can operate in many modes but the two principal modes are the normal (broadside) and the axial (end-fire) modes.

- **Normal mode:** In the normal mode of operation the field radiated by the antenna is maximum in a plane normal to the helix axis and minimum along its axis, as shown sketched in Fig. 4-7. To achieve the normal mode of operation, the dimensions of the helix are usually small compared to the wavelength (i.e., $NL_0 \ll \lambda_0$).

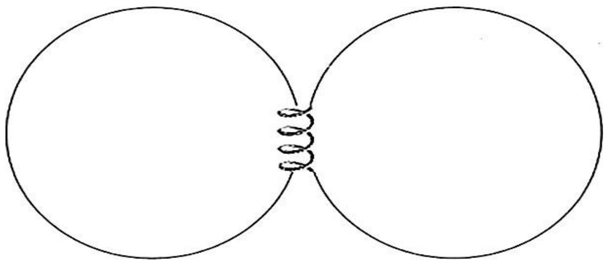


Fig. 4-7 Radiation pattern of helix normal mode

- In the normal mode, the helix of Fig. 4-8 (a) can be simulated approximately by N small loops and N short dipoles connected together in series as shown in Fig. 4-8 (b). The fields are obtained by superposition of the fields from these elemental radiators. The planes of the loops are parallel to each other and perpendicular to the axes of the vertical dipoles. The axes of the loops and dipoles coincide with the axis of the helix.

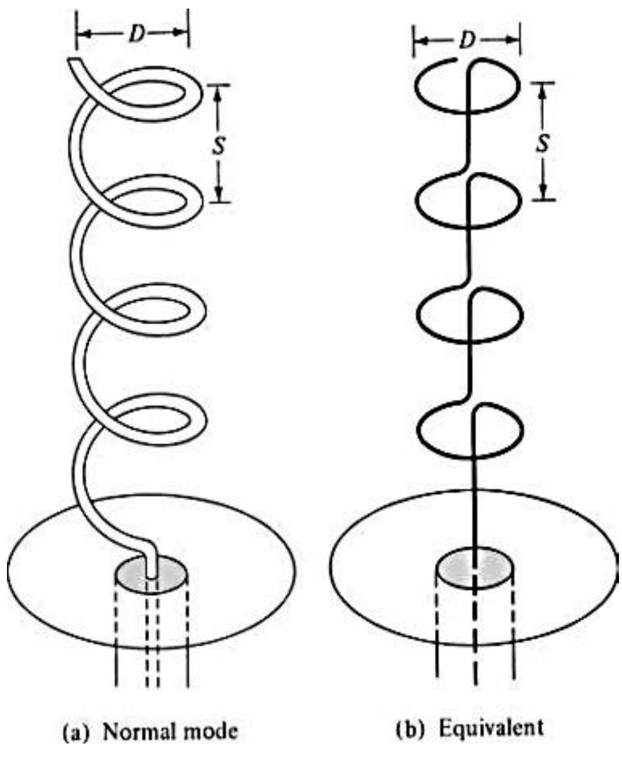


Fig. 4-8 Normal (broadside) mode for helical antenna and its equivalent

- Since in the normal mode the helix dimensions are small, the current throughout its length can be assumed to be constant and its relative far-field pattern to be independent of the number of loops and short dipoles.

- Thus its operation can be described accurately by the sum of the fields radiated by a small loop of radius D and a short dipole of length S, with its axis perpendicular to the plane of the loop, and each with the same constant current distribution.
- The far-zone electric field radiated by a short dipole of length S and constant current I_0 is $\boldsymbol{E_\theta}$, and it is given by,

$$\boldsymbol{E_\theta} = j\eta \frac{kI_0 S e^{-jkr}}{4\pi r} \sin\theta \qquad \text{-------- (4.21)}$$

where l is being replaced by S.

In addition the electric field radiated by a loop is $\boldsymbol{E_\phi}$, and it is given by,

$$\boldsymbol{E_\phi} = \eta \frac{k^2 (D/2)^2 I_0 e^{-jkr}}{4r} \sin\theta \qquad \text{-------- (4.22)}$$

- The ratio of the magnitudes of the $\boldsymbol{E_\theta}$ and $\boldsymbol{E_\phi}$ components is defined as the axial ratio (AR), and it is given by

$$\boldsymbol{AR} = \frac{|\boldsymbol{E_\theta}|}{|\boldsymbol{E_\phi}|} = \frac{4S}{\pi k D^2} = \frac{2\lambda S}{(\pi D)^2} \qquad \text{-------- (4.23)}$$

- By varying the D and/or S the axial ratio attains values of $0 \le AR \le \infty$. The value of $AR = 0$ is a special case and occurs when $\boldsymbol{E_\theta} = 0$ leading to a linearly polarized wave of horizontal polarization (the helix is a loop).
- When $AR = \infty$, $\boldsymbol{E_\phi} = 0$ and the radiated wave is linearly polarized with vertical polarization (the helix is a vertical dipole). Another special case is the one when AR is unity ($AR = 1$) , radiated wave is circularly polarized.

$$\frac{2\lambda_0 S}{(\pi D)^2} = 1 \qquad \text{-------- (4.24)}$$

$$C = \pi D = \sqrt{2S\lambda_0} \qquad \text{-------- (4.25)}$$

$$\tan\alpha = \frac{S}{\pi D} = \frac{\pi D}{2\lambda_0} \qquad \text{-------- (4.26)}$$

- Practically this mode of operation is limited and it is hardly used because its bandwidth and radiation efficiency is very small.
- **Axial Mode:** In this mode of operation, there is only one major lobe and its maximum radiation intensity is along the axis of the helix, as shown in Fig. 4-9. The minor lobes are at oblique angles to the axis. To excite this mode, the diameter D and spacing S must be large fractions of the wavelength.

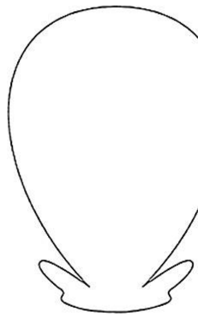


Fig. 4-9 Radiation pattern of helix axial mode

- Most often the antenna is used in conjunction with a ground plane, whose diameter is at least $\lambda_0/2$, and it is fed by a coaxial line. However, other types of feeds (such as waveguides and dielectric rods) are possible, especially at microwave frequencies. The dimensions of the helix for this mode of operation are not as critical, thus resulting in a greater bandwidth.

⇨ **Design Procedure:**

- The terminal impedance of a helix radiating in the axial mode is nearly resistive with values between 100 and 200 ohms.
- The input impedance (purely resistive) is obtained by ;

$$R \approx 140 \left(\frac{C}{\lambda_0}\right) \qquad \text{-------- (4.27)}$$

which is accurate to about ±20%, the half-power beamwidth by ;

$$HPBW\ (degrees) \approx \frac{52\ \lambda_0^{3/2}}{C\sqrt{NS}} \qquad \text{-------- (4.28)}$$

- The beamwidth between nulls ;

$$FNBW\ (degrees) \approx \frac{115\ \lambda_0^{3/2}}{C\sqrt{NS}} \qquad \text{-------- (4.29)}$$

- The directivity by

$$D_0\ (dimensionless) \approx 15\ N\ \frac{C^2 S}{\lambda_0^3} \qquad \text{-------- (4.30)}$$

- The axial ratio (for the condition of increased directivity) by

$$AR = \frac{2N+1}{2N} \qquad \text{-------- (4.31)}$$

- The normalized far-field pattern is given by

$$E = \sin\left(\frac{\pi}{2N}\right) \cos\theta\ \frac{\sin[(N/2)\psi]}{\sin[\psi/2]} \qquad \text{-------- (4.32)}$$

where

$$\psi = 2\pi \left[\frac{S}{\lambda_0}(1-\cos\theta) + \frac{1}{2N}\right] \qquad \text{-------- (4.33)}$$

- The above formula valid for , $12° < \alpha < 15°$, $N \geq 3$ and $\frac{3}{4} < \frac{C}{\lambda_0} < \frac{4}{3}$.

MODERN ANTENNAS – THE RECONFIGURABLE ANTENNA

- A reconfigurable antenna is an antenna capable of modifying dynamically its frequency and radiation properties in a controlled and reversible manner. In order to provide a dynamical response, reconfigurable antennas integrate an inner mechanism (such as RF switches, varactors, mechanical actuators or tunable materials) that enable the intentional redistribution of the RF currents over the antenna surface and produce reversible modifications over its properties.
- Reconfigurable antennas differ from smart antennas because the reconfiguration mechanism lies inside the antenna rather than in an external beamforming network. The reconfiguration capability of reconfigurable antennas is used to maximize the antenna performance in a changing scenario or to satisfy changing operating requirements.
- Reconfigurable antennas come in a large variety of different shapes and forms. Their operation can largely be analyzed through existing design principles by utilizing well defined antennas as the base design and a point of reference for the desired operation.
- By considering the properties of a base design, reconfigurable antennas can be classified according to three categories that describe their operation: (1) the reconfigurable antenna parameters of interest, (2) the proximity of reconfiguration, and (3) the continuity of reconfiguration (e.g., having reconfigurable antenna parameters over a continuous range of values).
- Reconfigurable antennas are typically described by the first of these categories, including reconfigurable radiation (pattern or polarization) and reconfigurable impedance (frequency or bandwidth).
- The proximity of reconfiguration describes physical properties inherent to the base antenna design—either direct (alteration of a driven element) or parasitic (alteration of a parasitic component).
- The continuity of the reconfiguration is defined by the nature and capabilities of the reconfiguration mechanism, either discrete (a finite number of reconfigured states) or continuous (reconfiguration within a range of states).

Frequency Reconfigurable Dipole:

- A generic wireless communication link shown in Fig. 4-10. , illustrates a basic application of reconfigurable antenna. This scenario involves a transmitter $\boldsymbol{T}$ that broadcasts to two sets of wireless receivers $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$. These receivers operate at two different frequency bands $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$, centered at $\boldsymbol{f}_1$and $\boldsymbol{f}_2$, respectively (with$\boldsymbol{f}_1 < \boldsymbol{f}_2$).

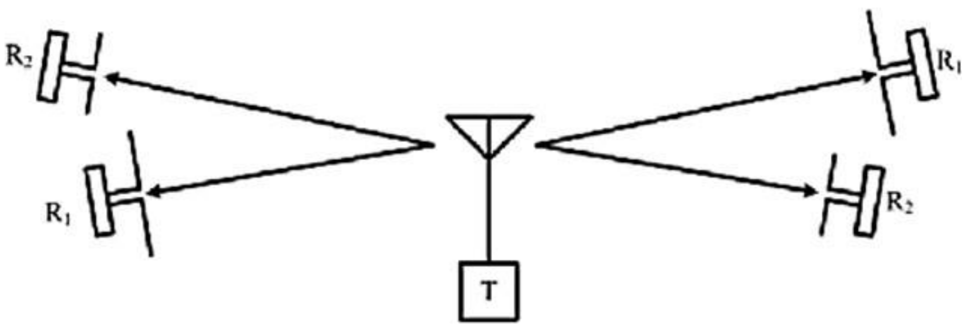


Fig. 4-10 Example of a wireless communication link utilizing reconfigurable antennas. The transmitter T utilizes a frequency reconfigurable antenna for broadcasting to the receivers R_1 and R_2, operating at f_1 and f_2, at time t_1 and t_2, respectively.

- The scenario assumes that all receiving antennas are coincidentally polarized, the transmitter broadcasts at the frequency bands centered at $f_1$and f_2, at times $t_1$and t_2,, respectively, and the radios require isolation between the bands such that a dual-band antenna is undesirable. Thus the reconfigurable antenna serves to allow communication with both sets of receivers using a single antenna.

ACTIVE ANTENNA

- The term *active antenna* implies an antenna integrated intimately with an active circuit, including the DC bias circuit, and without an isolator or circulator between them. The absence of isolator/ circulator implies that neither the antenna nor the circuit needs to be designed in a 50-Ω environment.
- Fig. 4-11 shows block diagrams of several types of active antennas, classified according to their functionality.

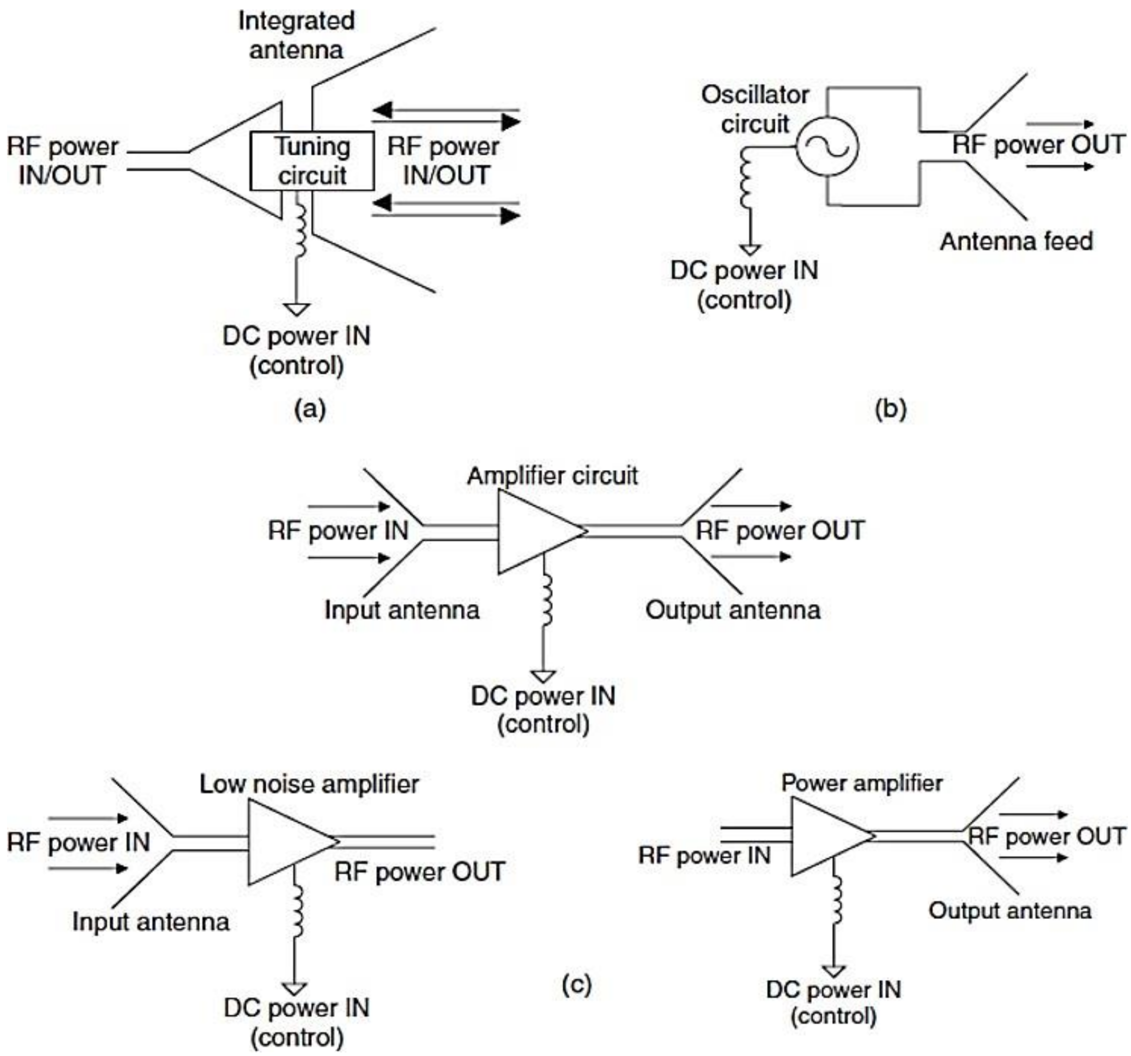


Fig. 4-11 Schematic diagrams of a few examples of active antennas: (*a*) actively tuned antenna, (*b*) oscillator antenna, and (*c*) amplifier antenna with repeater, receiver, and transmitter subclasses. An *active antenna* implies an antenna integrated intimately with an active circuit.

⇨ **A Classification of Active Antennas**

- Antennas are normally classified in terms of operational bandwidth (broadband and narrowband), implementation (e.g. printed, wire, or aperture), etc. In this chapter, it is appropriate to classify active antennas by the function of the active circuit.
- **Frequency-agile antennas** :
  - A two- or three-terminal active device can be designed into the antenna to enable the antenna impedance to tune with frequency.
  - Fig. 4-12 (a) shows an example of this type of active antenna: a slot microstrip feedline contains a varactor diode tuning element.
  - When the capacitance of the diode changes, the electrical length of the antenna, which in turn depends on the antenna reactance, changes and the antenna becomes resonant at a different frequency.
  - In this case, DC power is used to provide increased bandwidth of an antenna element. These antennas find applications in multifunctional systems with multiple nonsimultaneous carriers.

- **Oscillator antennas** :
  - A two- or three-terminal negative-resistance device can be connected directly to the terminals of a single antenna element or an array of elements.
  - In this case, DC power is converted to radiated RF power. An example of a patch antenna in the feedback loop of a transistor, shown in
  - Fig. 4-12 (b). Oscillator antennas have been discussed for applications such as low-cost sensors, power combining, and synchronized scanning antenna arrays.

- **Amplifier antennas**:
  - An active device is connected to the terminals of an antenna element to provide amplification in receive mode or transmit mode.
  - In the former case, the matching between the antenna and active element usually optimizes noise, while in the latter case, the matching optimizes power and/or efficiency.
  - Fig. 4-12 (c) shows an example of a repeater element with two slot antennas and a prematched amplifier chip. In this case, increase in gain is enabled by adding DC power to the antenna, and it becomes difficult to separate antenna gain from circuit gain.
  - Amplifier antennas find applications in transmitters where spatial power combining can be achieved with an array, and in receivers where the feedline loss, which contributes to the total noise figure, can be eliminated by directly connecting an LNA to the receiving antenna.

- **Frequency-conversion antennas**:
  - A two- or three-terminal active device integrated with an antenna can provide direct down or up conversion of a radiated signal, at frequencies that are direct harmonics or subharmonics of a fundamental frequency (multipliers, dividers), or at frequencies with a given offset from the operating frequency (mixers).
  - Fig. 4-12 (d) shows an example of a slot antenna with a Schottky diode, which can be used for frequency doubling since the slot is matched to the diode impedance at both the input frequency and its harmonic.
  - Such antennas have applications in receivers, mixers with high dynamic range, detectors for millimeter-wave and THz receivers, phase conjugating RFID type antennas, and high-frequency generation.
  - A special case of frequency-conversion antennas is when a two- or three-terminal rectifying device is connected directly to the terminals of a receiving antenna in such a way that the received RF power is converted with optimal efficiency to DC power, while harmonic production and re-radiation is minimized.
  - This type of active antenna is referred to as a rectenna. Such antennas have been applied to RFID tags, sensor powering for cases when there is no solar power and where it is difficult to replace batteries, directed narrow-beam array power beaming, and for energy recycling and/or scavenging.

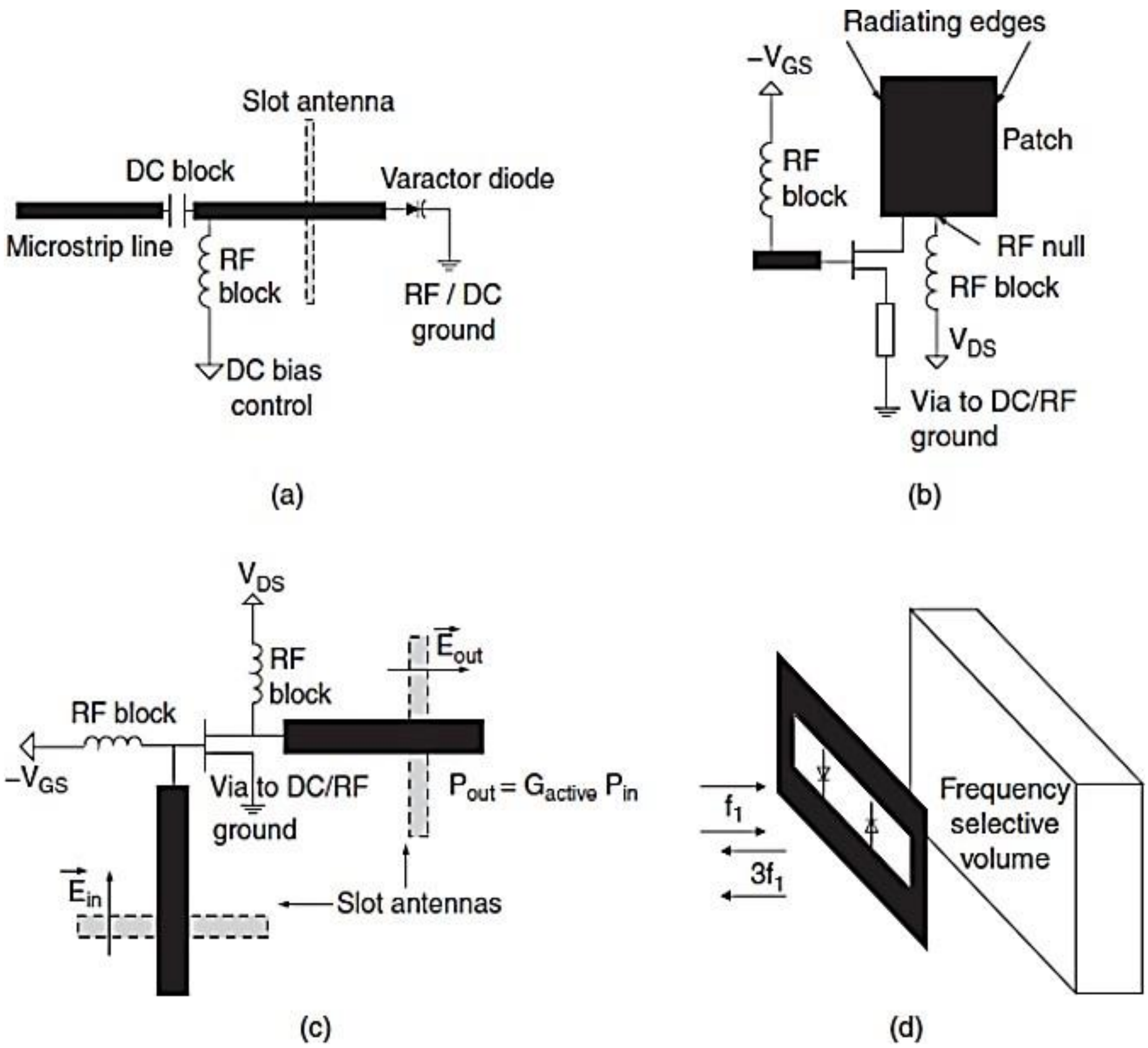


Fig. 4-12 Illustration of active antennas classified by their functionality: (*a*) a frequency-agile, in this case continuously tunable, slot antenna with a varactor diode active device; (*b*) an oscillator patch antenna loaded with a transistor; (*c*) a dual-slot amplifier repeater antenna; (*d*) a frequency-doubling slot antenna loaded with a Schottky diode

DIELECTRIC ANTENNAS

- The travelling wave antenna in which the travelling wave is guided by a dielectric is called dielectric antenna (Fig. 4-13).
- In dielectric antenna, near cut-off, the phase velocity rquals the velocity of light. The fields produced extend outside a dielectric guide.
- These outward fields excite the desired radiation in free-space. Such travelling wave antennas are useful for broad band signals.

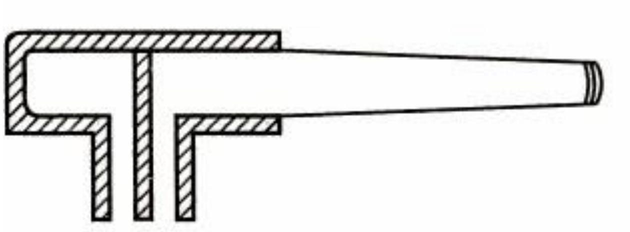


Fig. 4-13 Dielectric antenna

⇨ ***A dielectric resonator antenna :***

- **A dielectric resonator antenna** (DRA) is a radio antenna mostly used at microwave frequencies and higher, that consists of a block of ceramic material of various shapes, the dielectric resonator, mounted on a metal surface, a ground plane.
- Radio waves are introduced into the inside of the resonator material from thetransmitter circuit and bounce back and forth between the resonator walls, forming standing waves. The walls of the resonator are partially transparent to radio waves, allowing the radio power to radiate into space.
- An advantage of dielectric resonator antennas is they lack metal parts, which become lossy at high frequencies, dissipating energy. So these antennas can have lower losses and be more efficient than metal antennas at high microwave and millimeter wave frequencies.
- Dielectric waveguide antennas are used in some compact portable wireless devices, and military millimeter-wave radar equipment. The antenna was first proposed by Robert Richtmyer in 1939.
- In 1982, Long et al. did the first design and test of dielectric resonator antennas considering a leaky waveguide model assuming magnetic conductor model of the dielectric surface . Thus, they argued that the dielectric antenna behaved like a magnetic dipole antenna.
- The magnetic conductor model does not explain how current in the dielectric medium is transformed into electromagnetic waves which results in radiation. The electric field from oscillation of polarized dipole fall off inversely with cube of distance and cannot be responsible for far field radiation.

ELECTRONIC BAND GAP STRUCTURE (EBG) AND APPLICATIONS

- ***EBG definition*** : Periodic structures are abundant in nature, which have fascinated artists and scientists alike. When they interact with electromagnetic waves, exciting phenomena appear and amazing features result.
- In particular, characteristics such as frequency stop bands, pass bands, and band gaps could be identified. Reviewing the literature, one observes that various terminologies have been used depending on the domain of the applications.
- These applications are seen in filter designs, gratings, frequency selective surfaces (FSS), photonic crystals and photonic band gaps (PBG), etc. Generally speaking, electromagnetic band gap structures are defined as artificial periodic (or sometimes non-periodic) objects that prevent/assist the propagation of electromagnetic waves in a specified band of frequency for all incident angles and all polarization states.

- EBG structures are usually realized by periodic arrangement of dielectric materials and metallic conductors. In general, they can be categorized into three groups according to their geometric configuration: (1) three-dimensional volumetric structures, (2) two-dimensional planar surfaces, and (3) one-dimensional transmission lines.
- Different EBG structures : 3-D EBG structures (a woodpile structure consisting of square dielectric bars and a multi-layer metallic tripod array) , 2-D EBG surfaces (mushroom-like surface and a uni-planar design without vertical vias), one-dimensional EBG transmission line designs as in Fig. 4-14
- 2-D EBG surfaces has the advantages of low profile, light weight, and low fabrication cost, and are widely considered in antenna engineering.
- The planar electromagnetic band gap (EBG) surfaces exhibit distinctive electromagnetic properties with respect to incident electromagnetic waves:

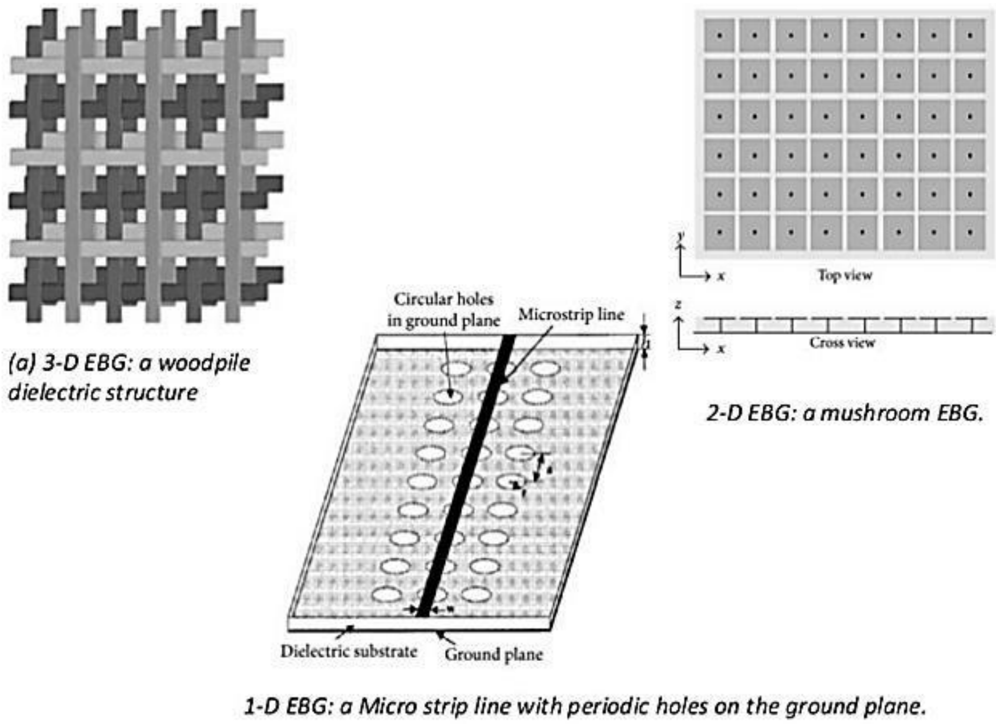


Fig. 4-14 Different EBG structures

⇨ **Applications:**

- A multitude of basic EBG applications exists especially within the microwave and low millimeter wave region. For example, In electronically scanned phased arrays, high-precision GPS, Bluetooth, mobile telephony, waveguides, antennas, low loss- coplanar lines, and compact integrated filters.

ANTENNA TEST RANGES

- Basically, there are two methods of antenna measurements: indoor and outdoor. Both the methods have their own limitations; the outdoor measurements are not protected from the environmental conditions whereas indoor measurements suffer space restrictions. In general, for the accurate measurements uniform plane waves should incident on the antenna and this is possible only if measurements are carried out in far-field region.
- The region in which antenna measurements are performed effectively is termed antenna ranges (separation between antennas) and it is basically of two types: reflection ranges and free-space ranges.
- ***Reflection ranges***: This is outdoor type test range, where ground is a reflecting surface. The reflection ranges create constructive interference in the region surrounding AUT which is referred as quit zone. In order to have effective communication height of Tx antenna is adjusted while that of Rx antenna is maintained constant. These testing ranges are found suitable for the antenna systems operating in frequency range from UHF to 16 GHz.
- ***Free-space ranges***: This is indoor type test range and is designed mainly to minimize environmental effects. This is most popular test range where the antennas are mounted over tall towers. The main problem of this method is reflection from the ground, which is reduced by
    - Selecting the directivity and SLL of the Tx antenna
    - Making LOS between antennas obstacle-free
    - Redirecting or absorbing reflected.
- Free-space ranges are further classified into elevated ranges, slant ranges, anechoic chambers and compact test ranges. Special indoor test ranges are near-field ranges, and they have several limitations compared to outdoor ranges.

⇨ **Anechoic Chamber Measurement:**

- Anechoic chamber is an indoor chamber. The chamber walls, celing and floor are filled with absorbing material except at the location of transmitting antenna and antenna under test (AUT).
- It simulates a reflection-less free space and allows all-weather antenna measurements in a controlled environment.
- As compared with outdoor ranges, in anechoic chamber, the area where test antenna situated is isolated from all types of interfering signals in better way. To improve isolation of test area, many times shielding is done which also allows Electromagnetic Compatibility (EMC) measurements.
- In case of small antennas, far field measurements is possible using anechoic chamber with one end wall opened and the chamber is combined with an outdoor range. But in case of large antennas, compact antenna test ranges and rear field ranges are installed in anechoic chambers itself. For such measurements a complete lining of absorbers is not essential.
- The absorbing materials are not only the integral part of measurement ranges but are the important components of antennas used to reduce the side lobe and back lobe radiations. Basically the typical broadband absorber used is carbon-loaded polyurethane foam. An ideal

absorber can provide an impedance match for the incoming waves at all frequencies and angles of incidence.

- By shaping absorber or by gradually varying resistivity of material, a tapered transition in impedance from free space to back of the absorber can be achieved. The most widely used geometrical shapers are pyramids and wedges as shown in Fig. 4-15 (a) and (b) respectively.

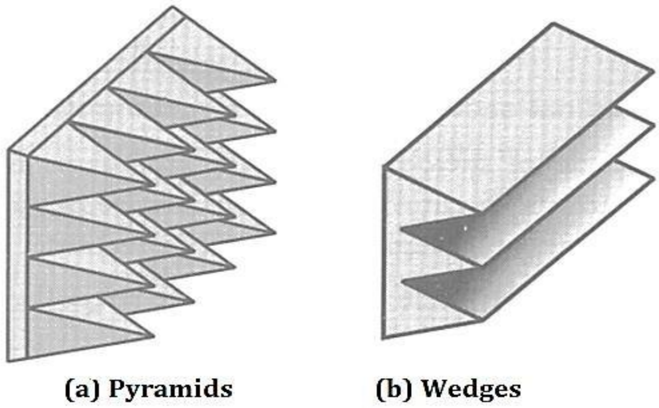


Fig. 4-15 Absorbing materials

- For normal incidence the pyramid type absorber is the best option as they scatter as a random rough surface when large compared with wavelength. At higher frequencies, the reflection coefficient is larger and at lower frequencies, the thickness of the absorber should be larger. Near gazing angler the pyramidal absorbers show large backscattered field.
- While the wedge shaped absorbers, with wedge direction along the plane of incidence, work perfectly at large angles of incidences but for normal incidence they cannot work satisfactorily compared with pyramidal absorbers.

⇨ **<u>Rectangular chambers:</u>**

- Fig. 4-16 shows a longitudinal sectional view of a rectangular chamber in which the source antenna is located at the centre of one of the end walls. The location of the test antenna is at a point approximately equidistant from the side and back walls along the centre line of the chamber at the other end of the chamber with respect to the source antenna.
- The chamber is completely lined with microwave absorbing material. Still there will be reflections from the walls, floor and ceiling and the specular reflections reaching .the test antenna are the cause of concern. These arise from the regions midway between the source and test antennas on the side walls, floor, ceiling and also from the centre region of the back wall.

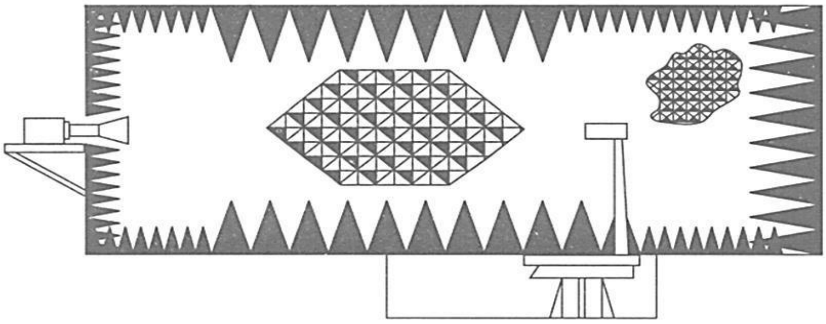


Fig. 4-16 A longitudinal sectional view of a rectangular anechoic chamber

- For good absorption by the lining materials, the chamber width and height is designed such that the angle of incidence $\theta_i < 60°$. However, this requirement puts the restriction that the length to width ratio of the chamber be about 2 : 1 which is extended to 3 : 1 sometimes at the expense of higher levels of reflections.
- The space in which the test antenna is located is termed the quiet zone. The volume of the quiet zone for a given chamber depends on the specified or allowable deviation of the incident field from a uniform plane wave. Rectangular chambers need bigger absorbing materials for frequencies below 1 GHz.
- Also at these frequencies it is difficult to obtain accurate measurements in these chambers mainly because it is usually not possible to obtain a source antenna with a sufficiently narrow beam width of these frequencies, to avoid illumination of the walls, floor and ceiling with the main beam.

⇨ **<u>Tapered chambers:</u>**

- The tapered anechoic chamber got introduced to overcome some of the limitations of the rectangular chamber, mentioned above. It consists of a tapered section opening into a rectangular section. The taper is shaped like a pyramidal horn that tapers from a small source end to a large rectangular test region. This construction is shown in Fig. 4-17.

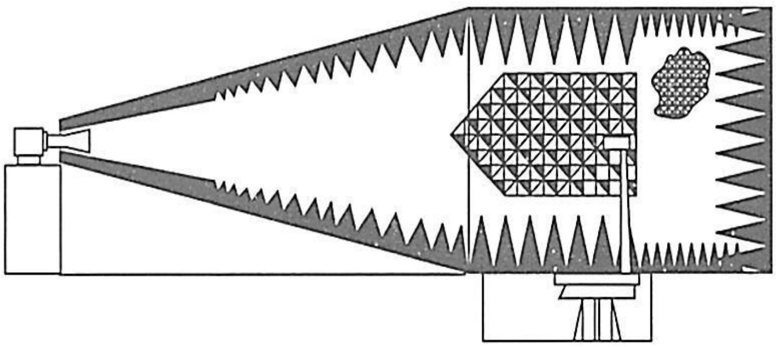


Fig. 4-17 A longitudinal sectional view of a tapered anechoic chamber

- The rectangular section is approximately which and the tapered section is usually twice as long as the rectangular section. This geometry inherently requires less absorbing material which helps in substantially reducing the cost. In the tapered chamber, the specular reflections that reach the test region occur close to the source antenna as shown in Fig. 4-35.

- However the path lengths of the reflected signals are not very different, electrically, from that of the direct signal which produces a slowly varying amplitude pattern which is beneficial since a constructive interference results. Also this allows use of thinner absorbing materials over the walls. The concept is illustrated in Fig. 4-18.

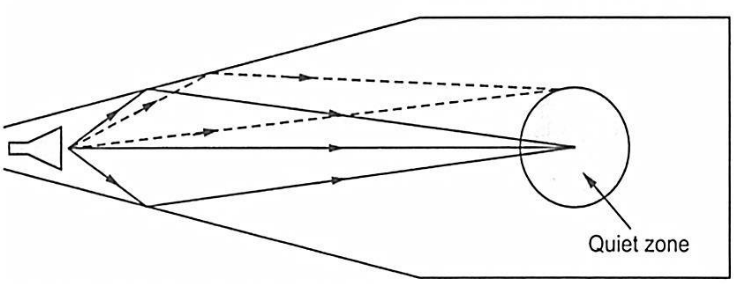


Fig. 4-18 Specular reflections that reach the quiet zone of a tapered anechoic chamber

MEASUREMENT OF RADIATION PATTERN OF ANTENNA:

- The radiation patterns (amplitude and phase), polarization, and gain of an antenna, which are used to characterize its radiationcapabilities, are measured onthe surface of a constant radius sphere.
- All these quantities are measured on the surface of a sphere with constant radius. Any point $'P'$ on such sphere can be described using spherical co-ordinate system as shown in Fig. 4-19.

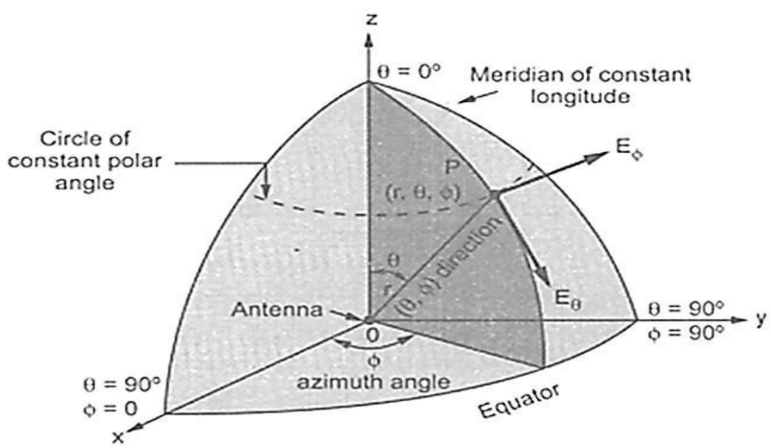


Fig. 4-19 Spherical co-ordinate system representation for radiation pattern measurement.

- Basically for representation of a point on the surface, only $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ specifications are sufficient because sphere with constant radius is considered. Thus the radiation characteristics of the antenna as a function of $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ for constant radius and frequency is called radiation pattern of an antenna.
- Basically it is a three dimensional representation. But due to the practical difficulty, number of two dimensional patterns are measured and from that the three dimensional pattern is constructed.

- In general, the minimum number of patterns required to construct a three dimensional pattern is 2 and they are selected as principle $\boldsymbol{E}$-plane and $\boldsymbol{H}$-plane patterns. The two dimensional pattern is generally called pattern cut.
- Generally the pattern cuts can be obtained for one of the angles ($\boldsymbol{\theta}$ or $\boldsymbol{\phi}$) constant and varying the other. In most of the cases, the required patterns are horizontal plane i.e. x-y plane and vertical pattern in x-z plane. (Refer Fig. 4-19.)
- For horizontal antenna following patterns are required.
  - The $\boldsymbol{\phi}$ component of electric field as a function of $\boldsymbol{\phi}$ is measured in x-y plane ($\boldsymbol{\theta} = 90°$). The field component can be then represented as $\boldsymbol{E_\phi}(\boldsymbol{\theta} = 90°, \boldsymbol{\phi})$ and it is called $\boldsymbol{E}$-plane pattern.
  - The $\boldsymbol{\phi}$ component of electric field as a function of $\boldsymbol{\theta}$ is measured in x-z plane ($\boldsymbol{\phi} = 0°$). It is represented as $\boldsymbol{E_\phi}(\boldsymbol{\theta}, \boldsymbol{\phi} = 0°)$ and it is called $\boldsymbol{H}$-plane pattern.
- The two patterns bisect the major lobe in mutually perpendicular planes providing sufficient information for the measurement.
- For vertical antenna following patterns are required.
  - The $\boldsymbol{\theta}$ component of electric field as a function of $\boldsymbol{\phi}$ is measured in x-y plane ($\boldsymbol{\theta} = 90°$). The field component can be then represented as $\boldsymbol{E_\phi}(\boldsymbol{\theta} = 90°, \boldsymbol{\phi})$ and it is called $\boldsymbol{H}$-plane pattern.
  - The $\boldsymbol{\theta}$ component of electric field as a function of $\boldsymbol{\theta}$ is measured in x-z plane ($\boldsymbol{\phi} = 90°$). It is represented as $\boldsymbol{E_\phi}(\boldsymbol{\theta}, \boldsymbol{\phi} = 90°)$ and it is called $\boldsymbol{E}$-plane pattern.
- For the antennas which are circularly or elliptically polarized, the measurement of all these four patterns is necessary. However the patterns in one plane provides sufficient information for the measurement.
- For example, for broadcasting applications and earth to earth communications, the horizontal plane patterns are sufficient. While for earth to space communications such as radar, radio astronomy etc., the vertical plane patterns are sufficient.
- The radiation pattern of an antenna can be measured either in transmitting mode or receiving mode. For reciprocal antennas, even any mode is sufficient, receiving mode is selected.

⇨ **<u>Basic Procedure for Radiation Pattern Measurement</u>**

- For the measurement of radiation pattern of antenna, two antennas are required. One of the antennas in the system is the antenna under test, while the other illuminates the antenna under test and it is located away from the antenna under test.
- Thus one antenna is used in the transmitting mode, while other in the receiving mode. But according to the reciprocity principle, the radiation pattern will be same irrespective of the mode in which antenna is used. The antenna under test is usually referred as primary antenna, while the other one as secondary antenna. Note that these are called primary or secondary antennas irrespective of the antenna mode i.e. either transmitting or receiving.

- The procedures for measuring the radiation pattern in a particular plane are as follows.
  - In the first procedure, the antenna under test i.e. primary antenna is kept stationary, while the secondary antenna is moved around the primary antenna along a circular path with uniform radius. If the secondary antenna is directional one, it is always aimed at the primary antenna. In this procedure, usually the primary antenna is transmitting. At different points, along the circular path, the readings of the field strength and direction with respect to the primary antenna are recorded. Then from these readings a plot of the radiation pattern of a primary antenna is plotted either as rectangular plot or polar plot.
  - In the second procedure, both the antennas are kept stationary with a suitable spacing between them. The secondary antenna is aimed at the primary antenna. The primary antenna is rotated about a vertical axis. In this procedure, the secondary antenna is used in the transmitting mode, so that the field strength reading and direction of the primary antenna with respect to the secondary antenna is made. The continuous readings at different points during rotation can be made using pattern recorder.
  - Generally at low frequency, first procedure is used while at high frequency second one is preferred.

⇨ **Set up for Measurement of Radiation Pattern of an Antenna**

- The simple arrangement for the radiation pattern measurement consists primary antenna is transmitting mode, secondary antenna as antenna under test. The secondary antenna is coupled with the rotating shaft and it is rotated using antenna rotator mechanism. To measure the relative amplitude of the received field an indicator is used along with the receiver as shown in Fig. 4-20.

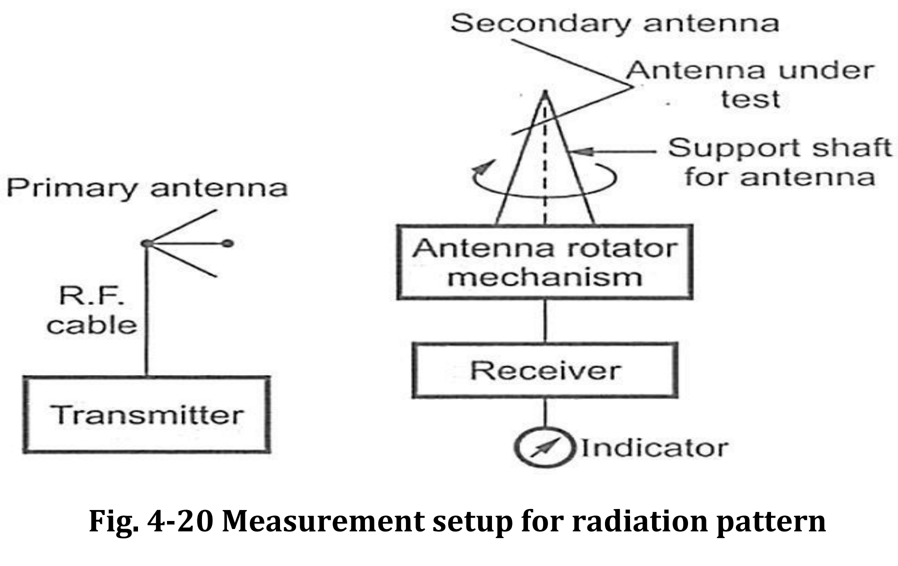


Fig. 4-20 Measurement setup for radiation pattern

- Usually the antenna under test is used in the receiving mode. It is properly illuminated by the stationary primary antenna. The secondary antenna is rotated about vertical axis. For E-plane pattern measurement, the antenna support shaft is rotated with both the antennas horizontal. While for H-plane pattern measurement, the shaft is rotated with both the antennas vertical.

GAIN MEASUREMENT:

- The gain and the directivity are usually measured in the direction of the pattern maximum. Their values in any other direction can be calculated from the radiation pattern. There are two techniques used for measuring the gain of an antenna-absolute gain measurement and gain transfer measurement.
- For the absolute gain measurement it is not necessary to have a prior knowledge of the gains of the antennas used in the measurement. The more commonly used gain transfer method, requires the use of a gain standard with which the gain of the antenna under test is compared.

⇨ **Absolute Gain Measurement**

- Friis transmission formula forms the basis for absolute gain measurement. The Friis transmission formula expressed in decibels is ;

$$P_{rdBm} = P_{tdBm} + G_{tdB} + G_{r\,dB} + 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) \quad \text{-------- (4.34)}$$

- Consider two identical antennas placed in an elevated range or inside a rectangular anechoic chamber which are properly oriented and aligned such that (i) they are polarization matched and (ii) main beams of the two antennas are aligned with each other. With this arrangement, the gain in the direction of the maximum can be measured. The gain in any other direction can be computed from the radiation pattern.
- Let R be the separation between the two antennas chosen such that the antennas operate in the far-field region. Let λ be the wavelength corresponding to the operating frequency. A calibrated coupling network and a matched receiver unit, as shown in Fig. 4-21, are used to measure the transmit and the receive powers P_{tdBm} and P_{rdBm} respectively. All the components are impedance matched using tuners.

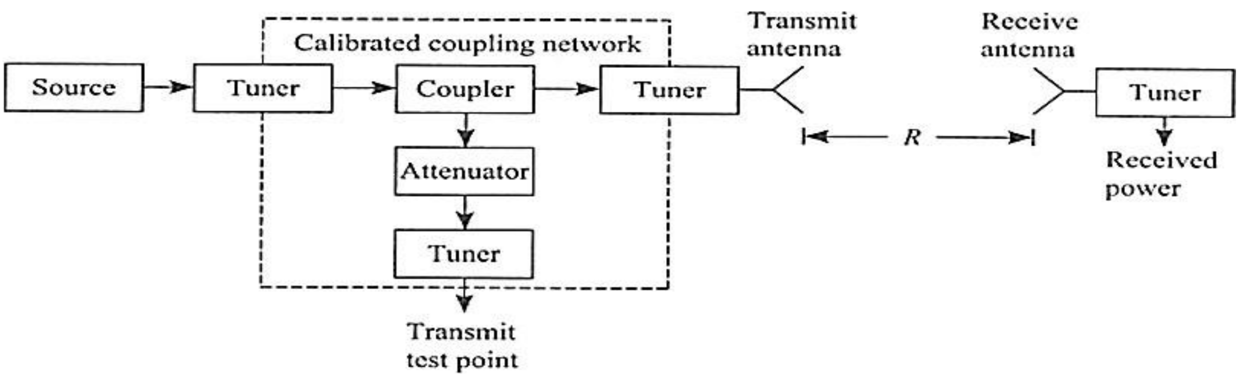


Fig. 4-21 Measurement of transmit and receive powers

- If the two antennas are identical, their gains are identical and Eqn. (4.34) can be written as

$$G_{tdB} = G_{r\,dB} = \frac{1}{2}\left[P_{rdBm} - P_{tdBm} - 20\log_{10}\left(\frac{\lambda}{4\pi R}\right)\right] \quad \text{-------- (4.35)}$$

and hence the gain of the antennas can be calculated. Since this method uses two antennas, it is known as *two-antenna method* for gain measurement.

- In the absence of two identical antennas, a third antenna is required to measure the gain. This is known as a *three-antenna method* of gain measurement.
- Let $G_A\,dB$, $G_B\,dB$, and $G_C\,dB$ be the gains of the three antennas. Transmitted and received powers are recorded by taking two antennas at a time. Let P^{AB}_{rdBm}, P^{BC}_{rdBm} and P^{CA}_{rdBm} be the received powers.
- The superscripts represent the antenna combinations used in the measurement. Similarly, let P^{AB}_{tdBm}, P^{BC}_{tdBm} and P^{CA}_{tdBm} be the transmitted powers in each measurement. Now we have three linear equations corresponding to these three measurements.

$$G_{AdB} + G_{BdB} = P^{AB}_{rdBm} - P^{AB}_{tdBm} - 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) \qquad \text{------}$$

$$G_{BdB} + G_{CdB} = P^{BC}_{rdBm} - P^{BC}_{tdBm} - 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) \qquad \text{------}$$

$$G_{AdB} + G_{CdB} = P^{CA}_{rdBm} - P^{CA}_{tdBm} - 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) \qquad \text{------}$$

which can be solved simultaneously to calculate the gains of each of the antennas.

⇨ Gain Transfer Method

- The gain of the test antenna is measured by comparing it with a standard gain antenna, of which the gain is known accurately. The test antenna is illuminated by a plane wave with its polarization matched to the test antenna. The received power into a matched load, P^{T}_{rdBm} is then measured. Let G^{T}_{dB} be the gain of the test antenna. From Friis formula ;

$$G_{tdB} + G^{T}_{dB} = P^{T}_{rdBm} - P_{tdBm} - 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) \qquad \text{--------} (4.39)$$

- Now the test antenna is replaced by a standard gain antenna having a gain of G^{S}_{dB} and the received power P^{S}_{rdBm} is measured. Again from Friis transmission formula

$$G_{tdB} + G^{S}_{dB} = P^{S}_{rdBm} - P_{tdBm} - 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) \qquad \text{--------} (4.40)$$

- Subtracting Eqn. (4.39) from (4.40) ;

$$G^{T}_{dB} = G^{S}_{dB} + P^{T}_{rdBm} - P^{S}_{rdBm} \qquad \text{--------} (4.41)$$

and hence the gain of the test antenna can be calculated.

- It is important to note that the polarization of the test antenna and the standard gain antenna need to be identical to each other and this should be matched with the polarization of the transmitter. Both antennas should be impedance matched to the receiver. This method is used to measure the gain of a linearly polarized antenna.
- The gain of a circularly polarized antenna can also be measured using a linearly polarized standard gain antenna. Since a circularly polarized wave can be decomposed into two orthogonal linear components, we can use a linearly polarized antenna to measure the gains of each of these components and then the total gain is obtained by combining the two.

- For this, we take a linearly polarized transmit antenna and orient it so that it produces vertically polarized waves. Now, the power received by a standard gain antenna oriented to receive vertical polarization and the power received by the circularly polarized test antenna is measured.
- The gain of the test antenna for the vertically polarized wave is computed using Eqn. (4.46). Let this gain be denoted by G_{dB}^{TV}. Now, rotate the transmit antenna by 90°, so that it radiates horizontally polarized waves.
- Measure the powers received by the test antenna and by the standard gain antenna oriented to receive horizontally c-arized waves (rotating it by 90°). Once again, using Eqn. (4.46) compute the gain, G_{dB}^{TH}, of the test antenna for the horizontally polarized wave. The total gain of the test antenna is given by ;

$$G_{dB}^{T} = 10 \log_{10}(G^{TV} + G^{TH}) \quad \text{-------- (4.42)}$$

- In the above equation, G^{TV} and G^{TH} are the gains expressed as ratios and not in decibels.

DIRECTIVITY MEASUREMENT

- Sometimes it is found that the directivity of the antenna cannot be calculated using the analytical techniques alone. So the directivity can be obtained from the radiation pattern of the antenna. The following procedure is considered ;
  - Measure the two principal E- and H-plane patterns of the test antenna.
  - Determine the half-power beamwidths (in degrees) of the E- and H-plane patterns.
  - Compute the directivity using the formula ;

$$D_0 = \frac{4\pi}{\Omega_A} \quad \text{-------- (4.43)}$$

$$\Omega_A = \theta_{HP}\,\phi_{HP} \quad \text{-------- (4.44)}$$

where θ_{HP} and ϕ_{HP} are the half-power beamwidths (HPBW) in the two principal planes,.

- Directivity can also be computed using the formula ;

$$D = \frac{4\pi U}{P_{rad}} \quad \text{-------- (4.45)}$$

where ; P_{rad} = Total power radiated in the direction (θ, ϕ)
$U(\theta, \phi)$ = Radiation intensity in the direction (θ, ϕ)

- The total power radiated by the antenna is obtained by integrating the radiation intensity over the entire solid angle of 4π. Thus,

$$P_{rad} = \oiint_{\Omega} U(\theta, \phi)\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\varphi \quad \text{-------- (4.46)}$$

POLARIZATION MEASUREMENT

- **Polarization Pattern Method :** This method can be used to measure the AR and the tilt angle τ of the polarization ellipse but not the sense of polarization (Fig. 4-22). The test antenna is connected as the source antenna while a linearly polarized antenna such as a dipole antenna is used to receive the power at different rotation angles.
- The square root of the received power plotted against the rotation angle ψ indicate the AR and title τ.

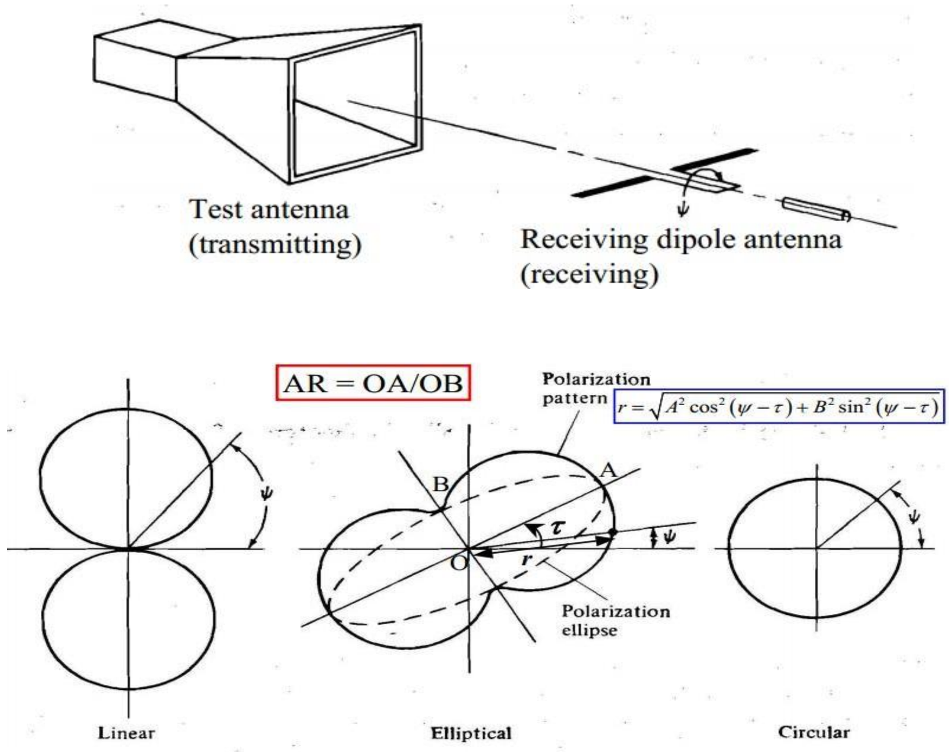


Fig. 4-22 Polarization Pattern Method

INPUT IMPEDANCE, REFLECTION COEFFICIENT, VSWR MEASUREMENT

- The input characteristics of an antenna such as the input impedance Z_A can be measured by a network analyser. The advantage of a network analyser is its ability to measure both the magnitude and the phase of the power received.
- Reflection Coefficient Measurement: The reflection coefficient ρ(or S_{11}) of an antenna can be obtained from its input impedance measurement.

$$\rho \text{ or } S_{11}=\frac{Z_A-Z_0}{Z_A+Z_0} \quad \text{(dimensionless)}$$

- VSWR Measurement: The VSWR of an antenna can be obtained from its reflection coefficient measurement.

$$\text{VSWR}=\frac{1+|\rho|}{1-|\rho|} \quad \text{(dimensionless)}$$

Post – MCQs:

1. ***Rumsey's principle***,
   a. which states that the impedance and pattern properties of an antenna will be frequency independent if the antenna shape is specified only in terms of angles.
   b. which states that the capacitance and pattern properties of an antenna will be frequency independent if the antenna shape is specified only in terms of angles.
   c. which states that the inductance and pattern properties of an antenna will be frequency independent if the antenna shape is specified only in terms of angles.
   d. None of the above

Ans: a

2. __________________ is a broadband antenna in which the geometry of the antenna structure is adjusted such that all the electrical properties of the antenna are repeated periodically with the logarithm of the frequency
   a. Helical Antenna
   b. Spiral Antenna
   c. Log periodic antenna
   d. Parabolic antenna

Ans: c

3. the relationship between the apex angle α, spacing s, and length l is
   a. tan α = $\left(1-\frac{1}{k}\right)/4S_{\lambda}$
   b. tan α = $\left(1-\frac{1}{t}\right)/4NY$
   c. tan α = $\left(1-\frac{1}{k}\right)/4Rm$
   d. None of the above

Ans: a

4. In the normal mode of operation of Helical Antenna, the dimensions of the helix should be _______
   a. $NL0 \ll \lambda 0$
   b. $NL1 \ll \lambda 0$
   c. $NL0 \ll \lambda 1$
   d. $NL0 \ll$ N0

Ans: a

5. The Oscillator antenna is defined as
   a. A two- or three-terminal negative-resistance device can be connected directly to the terminals of a single antenna element or an array of elements.
   b. A two- or three-terminal active device can be designed into the antenna to enable the antenna impedance to tune with frequency.
   c. An active device is connected to the terminals of an antenna element to provide amplification in receive mode or transmit mode
   d. None of the above

Ans: a

6. is a radio antenna mostly used at microwave frequencies and higher, that consists of a block of ceramic material of various shapes, the dielectric resonator, mounted on a metal surface, a ground plane.

   a. A resonator antenna
   b. A dielectric resonator antenna
   c. **A dielectric antenna**
   d. **A active antenna**

Ans: b

7. Friis transmission formula is _______________

   a. $PrdBm = PtdBm + GtdB + Gr\ dB + 20 \log 10\ (\lambda/ 4\pi R)$
   b. $PrdBm = PrdBm + GtdB + Gt\ dB + 20 \log 10\ (\lambda/ 4\pi r)$
   c. $PrdBm = PtdBm + GtdB + Gr\ dB + 20 \log 10\ (\lambda/ 4\pi z)$
   d. $PtdBm = PtdBm + GrdB + Gr\ dB + 20 \log 10\ (\lambda/ 4\pi R)$

Ans: a

8. The total gain of the test antenna in gain transfer method is _________
   a. $G_{dB}{}^{T} = 10 \log 10(G^{TV} + G^{TH})$
   b. $G_{dB}{}^{T} = 10 \log 10(G^{TH} + G^{TV})$
   c. $G_{dB}{}^{T} = 10 \log 10(D^{TV} + D^{TH})$
   d. $G_{dB}{}^{T} = 10 \log 10(G^{TV} + D^{TH})$

Ans: a

9. The directivity of the antenna is measured by
   a. $\Omega A = \theta HP \phi HP$
   b. $D = 4\pi U / Prad$
   c. All the above
   d. None of the above

Ans: c

10. The reflection coefficient ρ(or $S11$) of an antenna is __________
    a. $\rho = (ZA - Zo) * (ZA + Zo)$
    b. $\rho = (ZA - Zo)/(ZA + Zo)$
    c. $\rho = (Zo - ZA)/(ZA + Zo)$
    d. $\rho = (ZA + Zo)/(ZA - Zo)$

Ans: b

11. The antenna measurement ranges are ________
    a. Reflection ranges
    b. Free space ranges
    c. Refraction ranges
    d. Both A and B

Ans: d

12. _____________ is a room designed to completely absorb reflections of either sound or electromagnetic waves.
    a. Anechoic chamber (Ans)
    b. AUT
    c. EMC Measurement
    d. None of the above

Ans: a

13. In the radiation pattern measurement consists primary antenna is transmitting mode, secondary antenna as ______________
    a. Receiving mode
    b. Transmitting mode
    c. AUT (Ans)
    d. All the above.

Ans: c

14. In absolute gain measurement the Calibrated coupling network has following
    a. Coupler
    b. Attenuator
    c. Tuner
    d. All the above (Ans)

Ans:d

15. The advantage of a network analyser is its ability to measure both
    a. the magnitude and the phase of the power received (Ans)
    b. the amplitude and the frequency of the power received
    c. the magnitude and the frequency of the power received
    d. None of the above

Ans: a

Conclusion:

At the end of the topic, students will be able –

1) To understand the basic concept of Special Antennas and their radiation characteristics.

2) To understand the principle operation of Special Antennas and their applications.

3) To get exposure to different types Antenna Measurements

References:

1. Constantine.A. Balanis "Antenna Theory Analysis and Design", Wiley Student Edition, 4th Edition 2016.
2. Rajeswari Chatterjee, "Antenna Theory and Practice" Revised Second Edition New Age International Publishers, 2006.
3. S. Drabowitch, "Modern Antennas" Second Edition, Springer Publications, 2007
4.Robert S. Elliott "Antenna Theory and Design" Wiley Student Edition, 2006.

Assignments:

1. Develop the condition for frequency independence, construction, analysis and characteristics features of frequency independent antennas.
2. Explain about Helical Antenna and Spiral Antennas.
3. With neat schematic diagram, discuss the construction, principle and operation of a log periodic antenna.
4. Demonstrate the compact antenna test ranges and near field ranges with neat diagrams.
5. Explain in detail about – (i) Radiation Pattern Measurement (ii) Gain Measurement

Subject Name: Antennas & Propagation

Topic Name: **Propagation of Radio Waves**

(Unit – 5)

Syllabus / Propagation of Radio Waves:

1. Modes of propagation , Structure of atmosphere
2. Ground wave propagation, Tropospheric propagation , Duct propagation, Troposcatter propagation
3. Virtual height, critical frequency , Maximum usable frequency – Skip distance, Fading , Multi hop propagation

Aim and Objective:

- To give insight of basic Knowledge of Structure of Atmosphere
- To understand the concepts of Ground wave propagation, Tropospheric propagation , Duct propagation, Troposcatter propagation
- To study the basics of Virtual height, critical frequency , Maximum usable frequency – Skip distance, Fading , Multi hop propagation

Pre – Test MCQs:

1.Free space may be divided in to the following

a.Line of sight (LOS) propagation

b.Ground wave propagation

c.Sky wave propagation

d.All the above

Ans: d

2. The Stratosphere distance ________

A) 50 to 90 KM

B) 100 to 120km

C) 40 to 80 km

D) 100 -150 Km

Ans:a

3. The outer atmosphere G region is above _____

A) 500km

B) 400km

C) 350km

D) 300km

Ans: b

4. This region between the top of troposphere and the beginning of the stratosphere is called

A) inospause

B) Statospasue

C) tropopause.

D) None of the above

Ans: c

5) The region between (____ km to ____ km) above the earth's surface is called region of calm or stratosphere,

A) 10 km to 40 km

B) 20 km to 50 km

C) 05 km to 50 km

D) 20 km to 80 km

Ans: b

6) F layer is also called as ________

A) Appleton layer

B) Rough Layer

C) both a &b

D) Concentrated layer

Ans: a

7) The waves, which while traveling, glide over the earth's surface are called ______

A) ground wave

B) scattered wave
C) reflected wave
D) direct wave

Ans: a

8) ___________ is very important from the point of view of long distance radio communication

A) Space wave propagation
B) The sky wave propagation
C) ground wave propagation
D) None of the above

Ans: b

9) In sky wave propagation the signals are affected by _____

A) Interferences
B) Fading
C) Absorption
D) all the above

Ans: b

10. Fading is minimized by using the technique

a. Space diversity
b. Frequency diversity
c. both a and b
d. Phase diversity

Ans: c

Pre-requisite

- Basic knowledge of Atmospheric Structure.
- Basic Knowledge of different antennas and their characteristics.

UNIT V: PROPAGATION OF RADIO WAVES

Modes of propagation , Structure of atmosphere , Ground wave propagation , Tropospheric propagation , Duct propagation, Troposcatter propagation , Flat earth and Curved earth concept Sky wave propagation –Virtual height, critical frequency, Maximum usable frequency – Skip distance, Fading , Multi hop propagation.

INTRODUCTION – MODES OF PROPAGATION:

- The energy radiated from a transmitting antenna may reach the receiving antenna over any of many possible propagation paths, some of which are illustrated in Fig. 5-1.

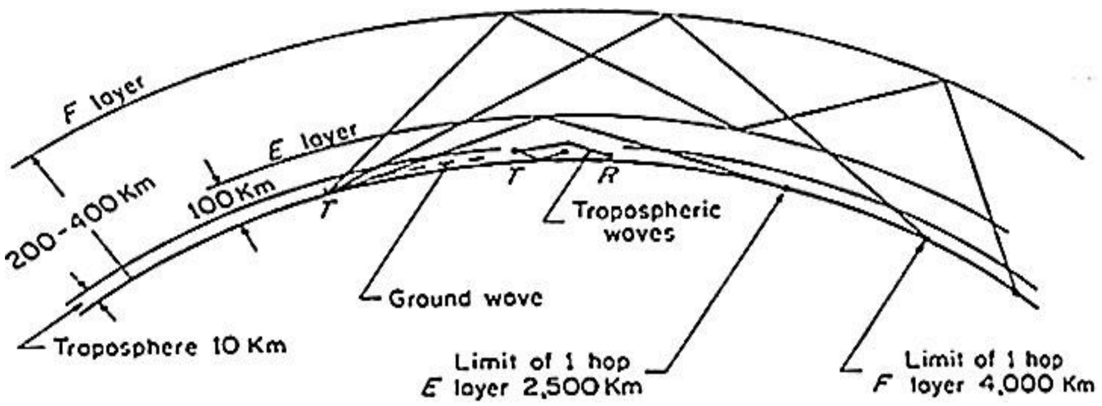


Fig. 5-1 Some possible propagation paths.

- Wavesthat arrive at the receiver after reflection or scattering in the ionosphere are known as *sky waves,* oralternatively, as *ionospherically reflected* and *ionospherically scattered waves.*
- Waves that are reflected or scattered in the troposphere (that region of the atmosphere within 10 kilometers of the earth's surface) are termed *tropospheric waves.*
- Energy propagated over other paths near the earth's surface is considered to be ground-wave. It is convenient to divide the ground-wave signal into the space wave and surface wave.
- The space wave is made up of the direct wave, the signal that travels the direct path from transmitter to receiver, and the ground-reflected wave, which is the signal arriving at the receiver after being reflected from the surface of the earth.
- The space wave also includes that portion of the energy received as a result of diffraction around the earth's surface and refraction in the upper atmosphere.
- The surface wave is a wave that is guided along the earth's surface, much as an electromagnetic wave is guided by a transmission line.
- Energy is abstracted from the surface wave to supply the losses in the ground; so the attenuation of this wave is directly affected by the constants of the earth along which it travels.
- When both antennas are located right at the earth's surface, the direct and ground-reflected terms in the space wave cancel each other, and transmission is entirely by means of this surface wave(assuming no sky wave or tropospheric wave).

FACTORS INVOLVED IN RADIO WAVE PROPAGATION

Radio wave or electromagnetic wave when travels from transmitter to receiver, many factors influence the propagation of wave. Some of the important factors are as follows:

- Characteristics of earth such as conductivity, permittivity, permeability.
- Curvature of the earth, magnetic field of the earth, roughness of the earth.
- Frequency of operation.
- Height and polarization of transmitting antenna and transmitter power.
- Characteristics of ionospheric regions.
- Distance between transmitter and receiver.
- Refractive index and permittivity of troposphere and ionospheric regions.

STRUCTURE OF ATMOSPHERE

- In the radio wave propagation, the earth's environment between the transmitting and receiving antennas play very important role. The atmosphere of the earth mainly consists of three regions namely (as in Fig. 5-2), i. Troposphere ii. Stratosphere and iii. Ionosphere
- In 1925, Sir Edward Appleton showed that propagation of the radio waves at high frequencies is greatly supported by the upper part of the atmosphere of the earth.

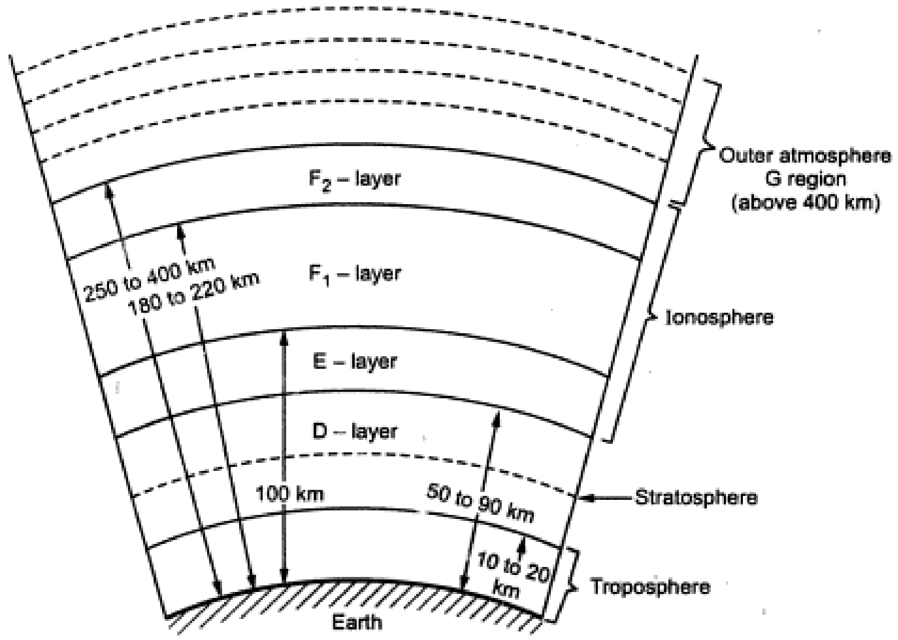


Fig. 5-2 Region in the earth's atmosphere

⇨ Structure of Troposphere:

- This is the nearest region in the atmosphere from the earth's surface and it is around 10 km to 20 km above the earth's surface. But the height of the troposphere region slightly varies at the poles and the equators. Its height is least at the poles while maximum at the equators.
- The gas components in the troposphere remain almost constant in percentage with increase in height. But the water vapour components drastically decrease with increasing height.

- The significant property of the tropospheric region is that temperature decreases with increase in the height. The troposphere is also called region of change. At a certain height called critical height above troposphere, the temperature remains constant for narrow region and then increases afterwards. This region between the top of troposphere and the begining of the stratosphere is called tropopause.

⇨ **<u>Structure of stratosphere:</u>**

- The region between 20 km to 50 km above the earth's surface is called region of calm or stratosphere. It is dense part of the atmosphere. It absorbs UV rays because of the presence of Ozone layer. The stratosphere has relatively little effect on radio waves because it is calm region with little or no temperature changes.

⇨ **<u>Structure of Ionosphere:</u>**

- The radiation from the space, in particular that from the sun, ionizes the gas molecules present in the atmosphere. The ionized layer that extends from about 50 km above the surface of the earth to several thousand kilometers is known as the ionosphere.
- At great heights from the surface of the earth the intensity of the ionizing radiation is very high, but there are very few molecules to be ionized. Therefore, in this region the ionization density (number of electrons or ions per unit volume) is low.
- As the height is decreased, atmospheric pressure increases, which implies that more molecules are present in the atmosphere. Therefore, the ionization density increases closer to the surface of the earth.
- With further reduction in height, though the number of molecules keeps increasing, the ionization density reduces because the energy in the ionizing radiation has been used up or absorbed to create ions.
- Therefore, the ionization density has a maximum that exists neither at the surface of the earth nor at the outer periphery of the ionosphere, but somewhere in the middle i.e., between 50 km to 400 km.
- It has been observed that the electron density profile (electron density versus height), has regions of maxima as well as regions of constant density (Fig. 5-3). These regions are known as layers of the ionosphere.
- There are mainly three layers in the ionosphere designated by the letters D, E, and F. The F layer splits into separate layers F_1 and F_2 during day time. The F layer is also called as Appleton layer and it is ionized during day time as well as night time. The E layer is also called as Kennelly-Heaviside layer.
- The D layer, which is present only during the day time, does not reflect high frequency electromagnetic waves ($2 - 30\ MHz$), but attenuates the waves passing through it. Even though the D layer reflects lower frequency waves ($< 1\ MHz$), due to the high absorption of the electromagnetic energy by the D layer, the utility of the reflected waves is limited.
- The E and F layers, which are present during both day and night times, make long distance communication possible by reflecting radio waves in the frequency range of $2–30\ MHz$. Radio waves above 30 MHz pass through the ionosphere.

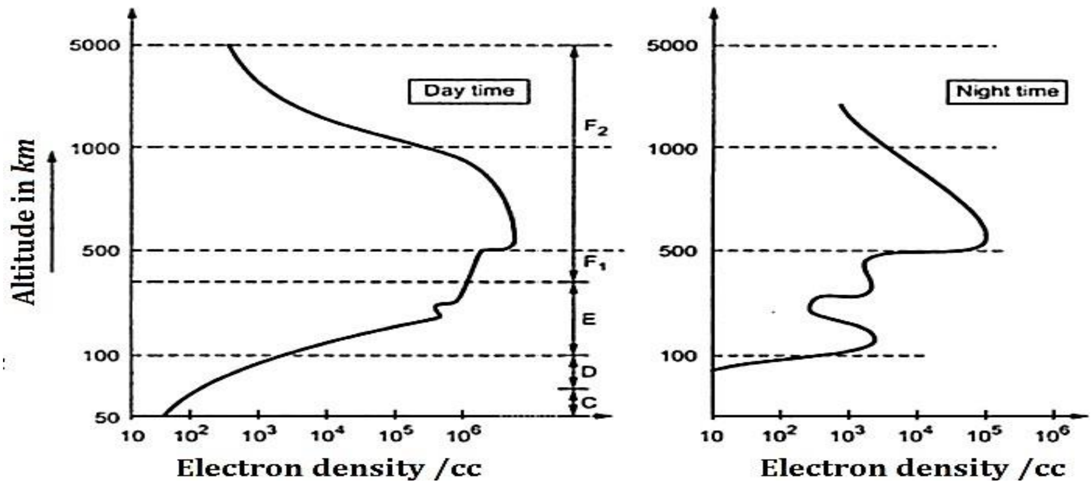


Fig. 5-3 Electron density profiles of ionosphere

- Along with the E layer, there exists the E_s layer which has very high ionization density. This is known as sporadic E layer and it exists during night time also. It is not important layer from the point of view of long distance communication. But it provides sometime better reception during night.
- The region lower to D region, where peak of the electron density is called C layer.
- The region at height 400 km above the earth's surface is called G region. Eventhough the upper limit of the ionosphere is not exactly known, the outer atmosphere is nothing but G region which consists of the charge particles trapped by the terrestrial magnetic field having shape similar to that of the magnetic lines of force. This region is occupied by the radiation belts girdling the earth.
- The number of layers in the ionosphere, their heights and the amount of sky wave that can bend by them will vary from day to day, month to month and year to year. For each layer there is a critical frequency, above which if radio wave is sent vertically upward, will not return back to the earth, but will penetrate it.

GROUND WAVE PROPAGATION

- The waves, which while traveling, glide over the earth's surface are called *ground waves.* The ground wave is also called *surface wave* as the wave passes over the surface of the earth. Ground waves are always vertically polarized (produced by vertical antennas) .
- The vertical antennas are the antennas in which the electromagnetic waves are vertically polarized i.e., electric field vectors of electromagnetic waves are vertical with respect to ground.
- Any horizontal component of the electric field vectors in contact with the ground gets short circuited.
- When the ground waves propagate along the surface of the earth, the charges are induced on the earth's surface.The number and polarity of these charges keep on changing with the intensity and location of the wave field. This variation causes the constitution of a current.

- In carrying this current, the earth behaves like a leaky capacitor. As the wave travels over the surface, it gets weakened due to absorption of some of its energy. This absorption, in fact, is the power loss in the earth's resistance due to the flow of current.
- This energy loss is partly replenished by the diffractionof energy, downward, from the portion of the wave present some what above the immediate surface of the earth. This process is shown in Fig. 5-4.

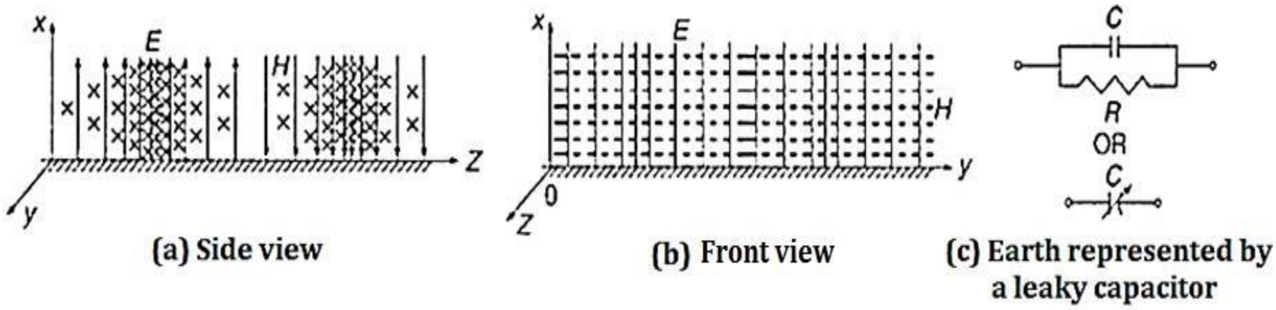


Fig. 5-4 Front and side view of the gliding wave and a leaky capacitor

- The earth's attenuation increases as frequency increases. So this mode of propagation is suitable for low and medium frequency i.e., upto 2 MHz only. It is also called as medium wave propagation. All the broadcast signals received during day time is due to ground wave propagation.
- Along with the ground attenuation, the ground waves or surface waves are suffered due to the diffraction and tilt in the wavefront. As the ground wave propagates over a surface of the earth, the wavefront gradually tilts more and more. As the wave front tilts more and more, the more electric field componentgets short circuited.
- Hence the strength of the signal gradually decreases with increase in the tilt. At a particular distance from the transmitter, the ground wave completely dies due to the attenuation as a result of more and more tilt of the wavefront.
- The phenomenon of wave tilting in successive wave front is shown in Fig. 5-5, in which T_1, T_2, T_3, T_4 and T_5 are the tilting angles in increasing order and W_1, W_2, W_3, W_4 and W_5 wavefronts.

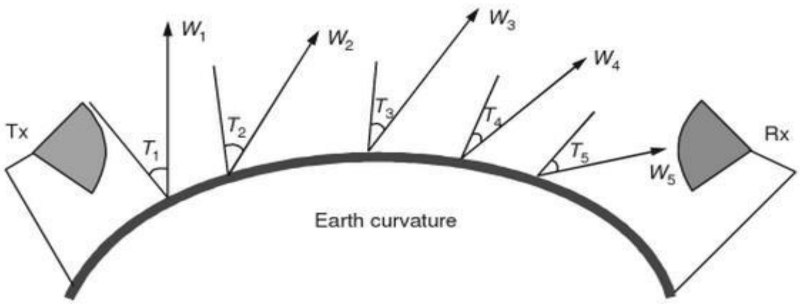


Fig. 5-5 Tilting wave fonts in ground wave propagation

- In general, surface of the earth is considered to be a plane if the distance between the transmitters and the receiver is less than the minimum barrier distance d given by expression ;

$$d = \frac{50}{(f_{MHz})^{\frac{1}{3}}} \qquad in\ miles \qquad \text{-------- (5.1)}$$

⇨ **Salient Features of Ground Wave Propagation**

Thesalient features of ground wave propagation are as given below:

- The ground waves propagate along the surface of the earth.
- When the ground waves propagate along the surface of the earth, the charges are induced on the surface of the earth. These charges travel along the wave and hence the current gets induced.
- While carrying induced current, the earth acts as a leaky capacitor.
- The ground waves are produced in vertically polarized antennas which are placedvery close to surface of the earth.
- The ground waves are important at broadcast and lower frequencies. These can beused upto 2 MHz.
- According to the characteristics of the earth, the strength of ground wave varies.These waves are not affected by the changes in the atmospheric conditions.
- The variations in surface or type of the earth affect propagation losses considerably.
- The maximum range of ground wave propagation depends -on the frequency andpower of the transmitter.

FREE SPACE PROPAGATION

- Free space implies an infinite space without any medium or objects that can interact with the electromagnetic waves.
- When electromagnetic waves are radiated by an antenna kept in free space, at large distances from the antenna the radiated fields are in the form of spherical waves with angular powerdistribution given by the antenna pattern.
- The power received, P_r, at a distance R is given by the Friis formula;

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} \qquad \text{-------- (5.2)}$$

where$P_t$is the transmit power and G_t and G_r are the transmit and receiveantenna gains, respectively.

- The transfer of electromagnetic energy from the transmit antenna to the receive antenna takes place in a straight-line path and, hence, such a communication link is called a line-of-sight (LOS) link.
- The factor $[\lambda/4\pi R]^2$ is due to the propagation and is called the free space path loss. It represents the attenuation of the signal due to the spreadingof the power as a function of distance, R. In decibel units, the path loss is expressed as ;

$$P_L = 10\log_{10}\left(\frac{4\pi R}{\lambda}\right)^2 \qquad \text{-------- (5.3)}$$

CALCULATION OF FIELD STRENGTH OF GROUND WAVE AT A DISTANCE

- The electric field strength, E at a distance from transmitting antenna due to ground wave can be obtained by solving Maxwell's equation and it is given ;

$$E = \frac{\eta\, h_t\, h_r I_s}{\lambda\, d} \quad \text{-------- (5.4)}$$

where ; η = Intrinsic impedance of free space ; h_t , h_r = Effective heights of transmitting and receiving antennas ; I_s = Antenna current and d =distance at a point from the transmitter.

- If d is large, the reduction in the field strength due to the ground attenuation and atmospheric absorption increases and thus actual voltage received at receiving point decreases.

SURFACE WAVE:

- Since the ground wave is guided along the surface of the earth, it is otherwise called as surface wave. Surface waves constitute the primary mode of propagation for frequencies in the range of a few kilohertz to several megahertz. Surface wave propagation exists when the transmitting and receiving antennas are close to the surface of the earth.
- For example, in the AM broadcast application, a vertical monopole above the ground is used to radiate power in the microwave frequency band. The receivers are generally placed very close to the surface of the earth and hence they receive the broadcast signals via surface waves. It is possible to achieve effective propagation over several hundred kilometers using the surface wave mode.
- The attenuation factor of the surface wave depends on the distance between the transmitter and the receiver, the frequency and the electrical properties of the ground (relative permittivity ϵ_r, and conductivity σ) over which the wave is propagating. At the surface of the earth, the attenuation factor is also known as the ground wave attenuation factor and is designated by A.

ATTENUATION CHARACTERISTICS FOR GROUND WAVE PROPAGTION

- According to **Sommerfield**, for a flat earth, the field strength for a ground wave propagation is given by ;

$$E_G = \frac{E_0 A}{d} \quad \text{-------- (5.5)}$$

where ; E_0 =Ground wave field strength at the earth's surface at unit distance (from the transmitting antenna) without considering earth losses ; E_G =Ground wave field strength ; A = Attenuation factor accounting for earth losses and d =distance from the transmitting antenna.

- The unit distance field strength E_0, depends on two factors, namely, power radiation of transmitting antenna and the diversity in the vertical and horizontal planes.
- For a vertical antenna which is non-directional in the horizontal plane, the radiation field produced is proportional to the cosine of angle of elevation. Then the field at unit distance is given by,

$$E_0 = \frac{300\sqrt{P}}{d} \quad \text{-------- (5.6)}$$

where ; P =Radiated power in kW

- The attenuation factor A depends on:
  i. Frequency, ii. Dielectric constant , iii. Conductivity of the earth
- For analysis purpose it is expressed interms of two auxillary variables such as, numerical distance p and phase constant b. These two variables are determined by frequency, distance and dielectric characteristics of the ground considered as conductor for radio frequency and are given as follows.
- The numerical distance p and phase constant b, are given by ;

$$p = \frac{\pi d}{\lambda_x} \cos b \; ; \; b = \tan^{-1}\left(\frac{\epsilon_r - 1}{x}\right)$$

$$x = \frac{\sigma}{\omega \epsilon_v} = \frac{18 \times 10^3 \sigma}{f_{MHz}}$$

- The relationship between the ground wave attenuation factor A and numerical distance p and phase constant b is shown in Fig. 5-6.

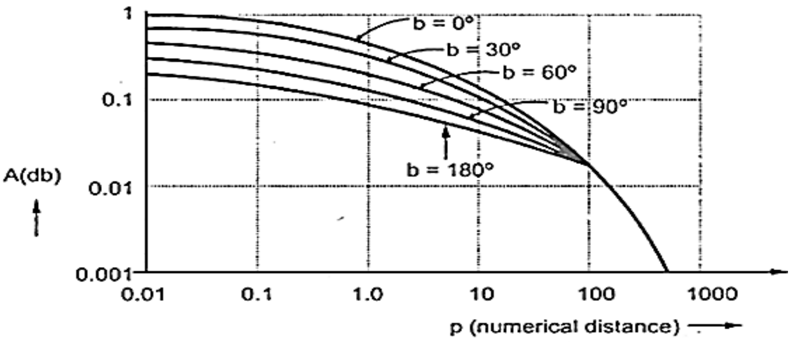


Fig. 5-6 Variation of A with numerical distance p for different values of the phase constant b.

- Note that the numerical distance p depends not only on frequency and ground constants but also on the actual distance to transmitter. It varies inversely with the ground conductivity, while it is proportional to the distance and square of frequency. Another parameter i.e. phase constant b is the measure of the power factor angle of the earth.
- The attenuation factor A may be approximately represented by following empirical formulae. For $b < 5°$;

$$A \approx \frac{2 + 0.3\,p}{2 + p + 0.6\,p^2} \qquad \text{-------- (5.7)}$$

- Similarly for all values of b, the attenuation factor A is approximately given by,

$$A \cong \left[\frac{2 + 0.3\,p}{2 + p + 0.6\,p^2}\right] - \sin b \sqrt{\frac{p}{2}}\, e^{-(5/8)p} \qquad \text{-------- (5.8)}$$

- For $x \gg \epsilon_r$, the power factor angle is nearly zero and the ground is mostly resistive. This is the case for average or better-than-average earth at broadcast frequencies.
- At very high frequencies and over poor earths the condition $\epsilon_r \gg x$ may be obtained, and the earth impedance will then be reactive.

- It will be noticed that the same earth which acts as a conductor at very low frequencies, will act as a dielectric that has a small loss at very high frequencies.
- From Fig. 5-6, the conclusions are ;
  - For $p < 1$: The ground attenuation factor A almost remains constant at unity and slowly reduces with increasing p. Then the ground losses are not significant for $p < 1$.
  - For $p > 1$: As the numerical distance p becomes greater than unity, the attenuation factor decreases rapidly.
  - For $p > 10$: For larger p , the ground attenuation factor is almost inversely proportional to the square of the distance.

SKY WAVE PROPAGATION

- The sky wave propagation is very important from the point of view of long distance radio communication.
- In this mode of propagation, the electromagnetic waves reaching the destination point first get reflected by the region of ionized gases in the upper atmosphere region which is situated between 50 km to 400 km above earth's surface. Hence, this mode of wave propagation is called ionospheric propagation.
- The ionosphere acts like a reflecting surface and reflect back the electromagnetic waves of frequencies between 2 MHz to 30 MHz. Electromagnetic waves of frequency more than 30 MHz are not reflected back from ionosphere but they penetrate it. This mode is most effective from the frequencies between 2 MHz to 30 MHz , hence this mode is also commonly called short wave propagation.
- With the sky wave propagation, a long distance point to point communication is possible. The main advantage of the sky wave propagation is that the long distance communication is possible with the help of multiple reflections of the sky waves. But these signals are affected by fading in which the strength of the signal varies with time.

⇨ Characteristics of Ionosphere

1. **Characteristics of *D* layer**
   - It is the lowest layer of the ionosphere and located at a height of 50 km to 90 km.
   - Its thickness is about 10 km.
   - It exists only in day-time and disappears in night time.
   - Its ionisation properties depend on the altitude of the sun above the horizon.
   - It is not useful layer for HF communication.
   - It reflects some VLF and LF waves.
   - It absorbs MF and HF waves to some extent.
   - Its electron density is 400 $electrons/cc$.
   - Crictical frequency of the layer is 100 kHz.

2. **Characteristics of *E* layer**
   - It exists next to D layer at an average height of 100 km.
   - Its thickness is about 25 km.

- It reflects some HF waves in day-time.
- Its electron density is $5 \times 10^5 electrons/cc$.
- Its critical frequency is 4 MHz.

3. **Characteristics of E_s layer**
    - It is a sporadic E-Layer.
    - Its appearance is sporadic in nature.
    - It exists in both day and night.
    - It is a thin layer and its ionisation density is high.
    - It appears close to E-Layer.
    - If it appears, it provides good reception.
    - It is not a dependable layer for communication.

4. **Characteristics of F_1 layer**
    - It exists at a height of about 180 km in day-time.
    - Its thickness is about 20 km.
    - It combines with F_2 layer during nights.
    - HF waves are reflected to some extent.
    - It absorbs HF to a considerable extent.
    - It passes on some HF waves towards F_2 layer.
    - Its critical frequency is 5 MHz.

5. **Characteristics of F_2 layer**
    - It is the most import layer for HF communication.
    - Its average height is about 325 km in day-time.
    - Its thickness is about 200 km.
    - It falls to a height of 300 km at nights as it combines with the F_1 layer.
    - It is the topmost layer of the ionosphere.
    - It is highly ionized and offers better HF reflection.
    - Electron density of F_2 layer is 2×10^6 $electrons/cc$.
    - Its critical frequency is 8 MHz in day time and 6 MHz at nights.

<u>IONOSPHERIC PROPAGATION</u> (Propagation of Radio Waves through Ionosphere)

- The sky wave propagation or ionospheric wave propagation is important as it assists global short wave communication. Due to the existence of the different ionized layers in the ionosphere, the long distance communication is possible.
- In ionosphere, the composition of the layers and heights of the layers vary with time, but the E and F layer persists permanently. These layers are mainly useful in long distance communication.
- The D-layer exists during day time. It cannot reflect high frequency waves back to the earth. Instead the intensity of the waves reflected back from the E or F layers decrease during day time due to the presence of the D-layer.
- The layers which exist permanently act as a radio mirror to bounce back the sky waves to the earth. The waves which return back to the earth appear to be the waves reflected by the layers of the ionosphere.

- But practically the ionized layers refract or bend the waves back towards the earth in much the same way as the refraction of the light waves travelling through media of different densities.
- The refraction mechanism can be explained in this fashion. When the wave approaches the ionized layer at an angle, the refractive index decreases as the ionization density increases.

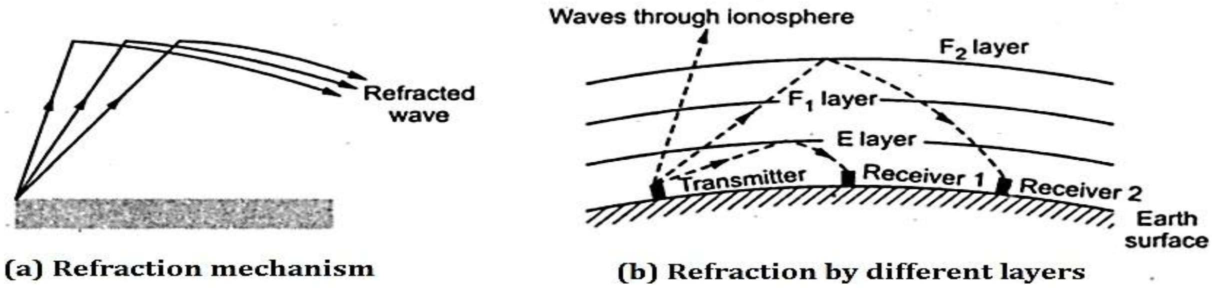


Fig. 5-7 Propagtion through ionosphere

- Hence the incident wave bends gradually further and further away from the normal as shown in the Fig. 5-7 (a). If the rate of change of the refractive index is sufficient, the refracted wave becomes parallel to the layer first, then it bends downward and then comes out of the ionized layer at an angle of incidence. The propagation of radio waves through ionosphere is as shown in the Fig. 5-7 (b).

CHARACTERISTIC PARAMETER OF IONOSPHERIC PROPAGATION

⇨ Critical Frequency

- *The critical frequency for the ionized layer of the ionosphere is defined as the highest frequency that can be reflected back to the earth by a particular layer for a vertical incidence.* It is denoted by f_{cr}. Note that the critical frequency is different for different layers.
- Let us assume that the ionosphere is lossless, has a relative permeability of unity and can be modeled as plane stratified media (Fig. 5-8).

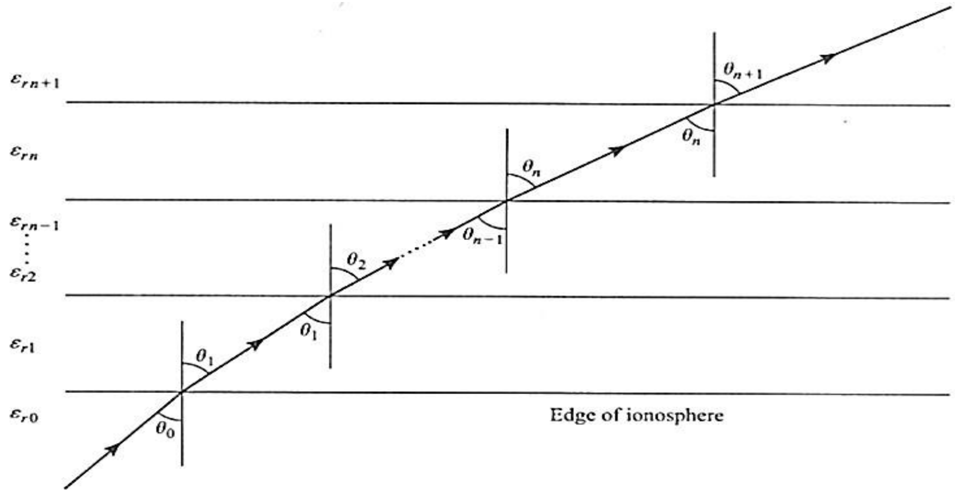


Fig. 5-12 Ray path through a plane stratified ionosphere

- The wavepath can be predicted using Snell's law

$$\sqrt{\epsilon_{r0}}\sin\theta_0 = \sqrt{\epsilon_{r1}}\sin\theta_1 = \cdots\cdots\cdots = \sqrt{\epsilon_{rn}}\sin\theta_n \qquad \text{-------- (5.9)}$$

  where θ_0 is the angle of incidence with respect to the normal and $\theta_1$is the angle of refraction at the lower edge of the ionosphere.

- At the next interface between layers having dielectric constants ϵ_{r1} and ϵ_{r2}, the angle of incidence is $\theta_1$and the angle of refraction is θ_2. At the lower edge of the ionosphere the electron density is zero and hence ϵ_{r0}=1. Therefore, the equation representing Snell's law reduces to ;

$$\sin\theta_0 = \sqrt{\epsilon_{rn}}\sin\theta_n \qquad \text{-------- (5.10)}$$

- The relative dielectric constant is a function of the electron density N. As the electromagnetic wave propagates deeper into the ionosphere, it passes through a region of higher N into a region of lower N.

- For a given angle of incidence $\theta_0 = \theta_i$, if N increases to a level such that the angle of refraction, $\theta_n = 90°$, the wave becomes horizontal. Under this condition Eqn. (5.10) reduces to

$$\sin\theta_i = \sqrt{\epsilon_{rn}} \qquad \text{-------- (5.11)}$$

- Let the dielectric constant of the nth layer be $\epsilon_{rn} = \epsilon_r$. Therefore , the refractive index of ionosphere can be defined by

$$\mathrm{n} = \sqrt{\epsilon_r} = [1 - (\frac{81\mathrm{N}}{f^2})]^{\frac{1}{2}} \qquad \text{-------- (5.12)}$$

  Substituting the value of ϵ_r from Eqn.(5.12)

$$\sin\theta_i = \sqrt{1 - \frac{81\mathrm{N}}{f^2}} \qquad \text{-------- (5.13)}$$

- If the incidence angle is greater than θ_i, the wave returns to the earth. For a given angle of incidence, higher frequency electromagnetic waves are reflected from the region having a higher value of N.

- Consider an electromagnetic wave launched vertically into the ionosphere having a maximum electron density N_{max}. Substituting $\theta_i = 0$ in Eqn. (5.13), the highest frequency that gets reflected is given by

$$f_{cr} = \sqrt{81 N_{max}} \qquad \text{-------- (5.14)}$$

  which is known as the critical frequency.

⇨ **<u>Virtual Height</u>**

- Consider an electromagnetic wave from a transmitter reaching the receiver after being reflected by the ionosphere as shown in Fig. 5-13. Let the wave enter the ionosphere at L, and take a curved path LMN before it emerges out of the ionosphere.

- The height at a point above the surface at which the wave bends down to the earth is called *actual height* or *true height.*
- If the incident and the reflected rays are extended, they meet at point O as shown in Fig. 5-13, i.e., it is more convenient to think of the wave being reflected rather than refracted. So the path can be assumed to be straight lines.
- The vertical height from the ground to the point O is known as the virtual height of the ionized layer and it is not true height.
- An ionosonde is the instrument used to measure the virtual height of the ionosphere. This instrument transmits an RF pulse vertically into the ionosphere from the ground. This pulse is reflected from the ionosphere and is received by the ionosonde. The time delay between the transmit and the receive pulse is measured and plotted as a function frequency of the electromagnetic wave.
- The time T duration required for the round trip is noted and then virtual height is determined by using ;

$$h = \frac{c\,T}{2} \qquad \text{-------- (5.15)}$$

where ; $h =$ virtual height , $c =$ velocity of light (m/s) and $T =$ time period (s).

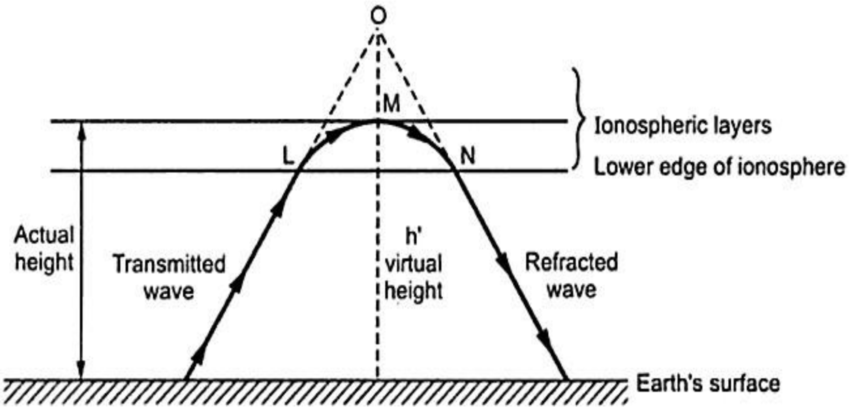


Fig. 5-13 Propagation path in the ionosphere and the depiction of virtual height

- The time delay is a measure of the virtual height of the ionosphere. A plot of the virtual height as a function of frequency is known as an ionogram. A typical ionogram for daytime is shown in Fig. 5-14.
- As the frequency of the electromagnetic wave increases, the virtual height also increases slightly, indicating that the waves of higher frequencies are returned from higher levels within the layer.
- As the frequency approaches the critical frequency (5 MHz for the F_1layer), the virtual height steeply increases. Once the critical frequency is crossed, the virtual height drops back to a steady value (350 km for 5.5 MHz) which is higher than that for a lower frequency (200 km for $4\ MHz$).

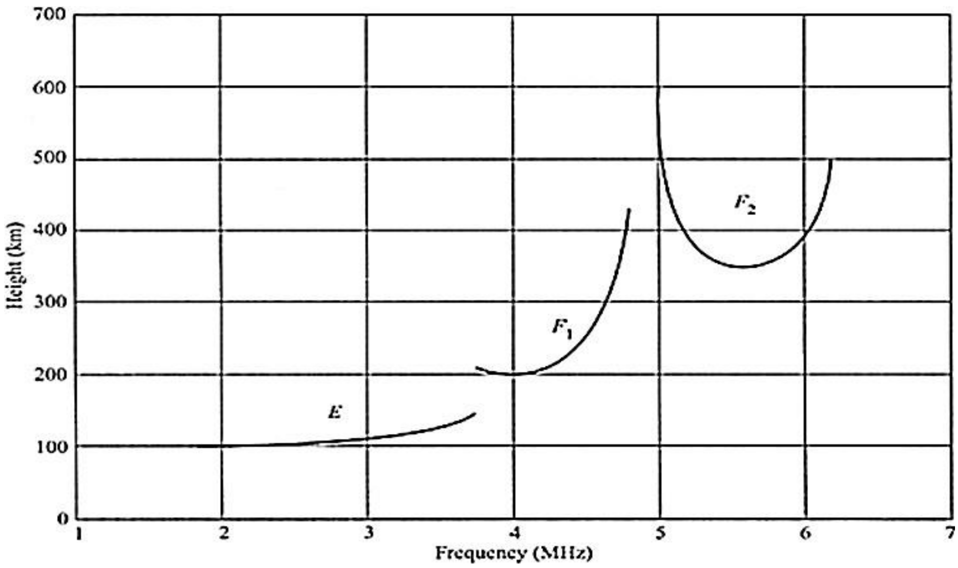


Fig. 5-14 A typical ionogram recorded during daytime

⇨ **Maximum Usable Frequency (MUF)**

- The critical frequency is the maximum frequency that can be reflected back to the earth by the ionosphere for the vertical incidence. But if the frequency of the radio wave exceeds the critical frequency f_{cr}, then the path of propagation in the ionosphere layer depends on the angle of incidence.
- Hence the maximum usable frequency is defined as the limiting maximum frequency that can be reflected back to the earth by the ionospheric layer for a specific angle of incidence other than the angle of incidence for vertical incidence. It is denoted by f_{MUF}.
- The maximum usable frequency f_{MUF} can also be defined as the maximum frequency that can be used for the sky wave propagation for specific distance between two points on the earth.
- Thus f_{MUF} is the highest frequency used for the sky wave communication and for each pair of points on the globe, the value of f_{MUF} will be different. Generally the value of f_{MUF} ranges between 8 MHz to 35 MHz.
- For any other angle of incidence, the highest frequency that can be reflected from the ionosphere will be greater than the critical frequency. The highest frequency that gets reflected by the ionosphere for a given value of angle of critical incidence (say θ_m), is known as the maximum usable frequency, f_{MUF}.

Substituting $\theta_i = \theta_m$ and $f = f_{MUF}$ in Eqn. (5.13) ;

$$\sin\theta_m = \sqrt{1 - \frac{81N_{max}}{f_{MUF}^2}} \quad \text{-------- (5.16)}$$

- From Eqn. (5.14), $81N_{max} = f^2{}_{cr}$ Substituting this in Eqn.(5.16);

$$\sin\theta_m = \sqrt{1 - \frac{f_{cr}^2}{f_{MUF}^2}} \quad \text{-------- (5.17)}$$

which can be written as ;

$$1 - \sin^2\theta_m = \frac{f_{cr}^2}{f_{MUF}^2} \quad \text{-------- (5.18)}$$

$$\cos^2\theta_m = \frac{f_{cr}^2}{f_{MUF}^2} \quad \text{-------- (5.19)}$$

- The expression that relates the critical frequency and the angle of incidence to the maximum usable frequency ;

$$f_{MUF} = f_{cr} \sec\theta_m \quad \text{-------- (5.20)}$$

- For example, if the critical frequency is 9 MHz ,the maximum usable frequency corresponding to an angle of incidence of 45° is 12.73 MHz.

⇨ **Skip Distance**

- *The skip distance is the shortest distance from the transmitter, measured along the surface of the earth, at which a sky wave of fixed frequency will return back to the earth.*
- The angle of incidence for which the wave returns back to the earth at minimum distance from the transmitter , i.e., at the skip distance is called angle of critical incidence.
- Assume that the ionosphere can be modeled as a flat reflecting surface at a height h (virtual height) from the surface of the flat earth.

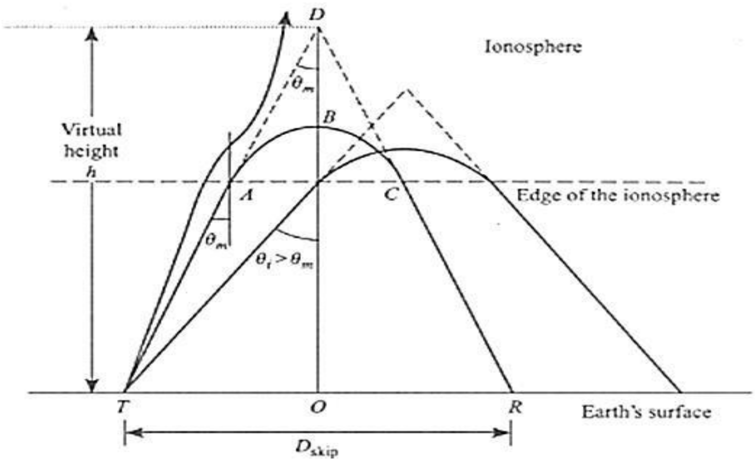


Fig. 5-15 Ray paths for different amgles of incidence, illustrating skip distance

- Consider the frequency of the transmitted wave is kept constant and the angle of critical incidence, θ_m ,.
- For launch angles, $\theta_i < \theta_m$, the waves are received beyond point R. For $\theta_i > \theta_m$, the ionosphere cannot reflect the waves back .
- Let the wave launched at $\theta_i = \theta_m$ reach the surface of the earth at R, at a distance of D_{skip} from the transmitter. The distance D_{skip} is known as the skip distance.
- In the region of radius less than D_{skip}, it is not possible to establish a communication link by the waves reflected from the ionosphere.

- To derive an expression for the skip distance in terms of the critical frequency and the maximum usable frequency by considering the ΔDOT in Fig. 5.15.
- From ΔDOT,

$$\cos^2\theta_m = \left(\frac{DO}{DT}\right)^2 = \left[\frac{h}{\sqrt{h^2 + \left(\frac{D_{skip}}{2}\right)^2}}\right]^2 = \frac{1}{1 + \left(\frac{D_{skip}}{2h}\right)^2} \qquad \text{-------- (5.21)}$$

- From Eqn. (5.19) and Eqn. (5.21) ;

$$D_{skip} = 2h\sqrt{\left(\frac{f_{MUF}}{f_{cr}}\right)^2 - 1} \qquad \text{-------- (5.22)}$$

⇨ **Optimum Working Frequency (OWF)**

- For the ionospheric propagation, it is desirable to use as high a frequency as possible. This clearly points out that the frequency used for the ionospheric transmission should be the maximum usable frequency i.e. MUF.
- But MUF depends upon the distance between the transmitter and the receiver and also upon the state of ionosphere. It is observed that due to the daily continuous changes and irregularities in the ionosphere, the MUF varies about 15% of its maximum value. Hence practically the frequency used should be 15% less than the value of MUF.
- Thus the frequency normally used for the ionospheric propagation is known as optimum working frequency. The optimum working frequency between the transmitter and the receiver for the ionospheric transmission is defined as the frequency laying between 50% to 85 % of the predicted MUF between the transmission and the reception points.
- It is observed that the maximum usable frequency at a particular location varies considerably with time of the day, from season to season and from months to months.

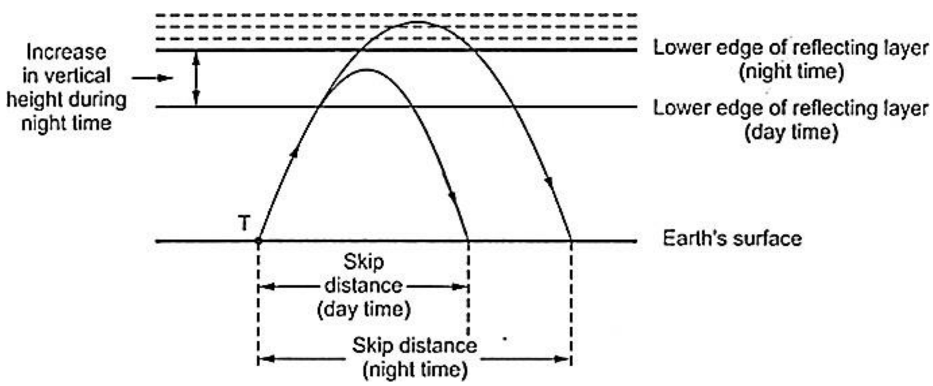


Fig. 5-16 Illustration of increase in vertical height and skip distance during night time.

- As the optimum working frequency is selected as the fraction of the maximum usable frequency, the OWF also varies in the similar way as the maximum usable frequency varies.

- Practically it is not at all possible to change the frequency of the signal propagated from hour to hour. Hence for the propagation ofwave, two frequencies are used namely one for the day time, while other for the night time.
- Sometimes it is preferred to have a third frequency even during the transition period from the day time to night time. It is observed that in the night time vertical height of the ionospheric layer increases as compared to that during the day time. Thus the skip distance also increases. It is illustrated in the Fig. 5-16.
- As we have studied that, the wave with lower frequency is bent more quickly as compared to the wave with higher frequency. Hence the increase in the skip distance during night time is cancelled by using lower frequency during night time.

MULTI-HOP PROPAGATION

- Let us now consider the transmit and receive antennas located on the surface of the spherical earth. The ionosphere is modelled as a spherical reflecting surface at a virtual height h from the surface of the earth [Fig. 5.17 (a)].
- A wave launched at a grazing angle ψ from point A gets reflected by the ionosphere (provided $\theta_i > \theta_m$) and reaches the surface of the earth at C.
- If the earth is a good reflector, the wave can undergo multi-hops and thus can establish communication between the points A and E in addition to that between A and C.

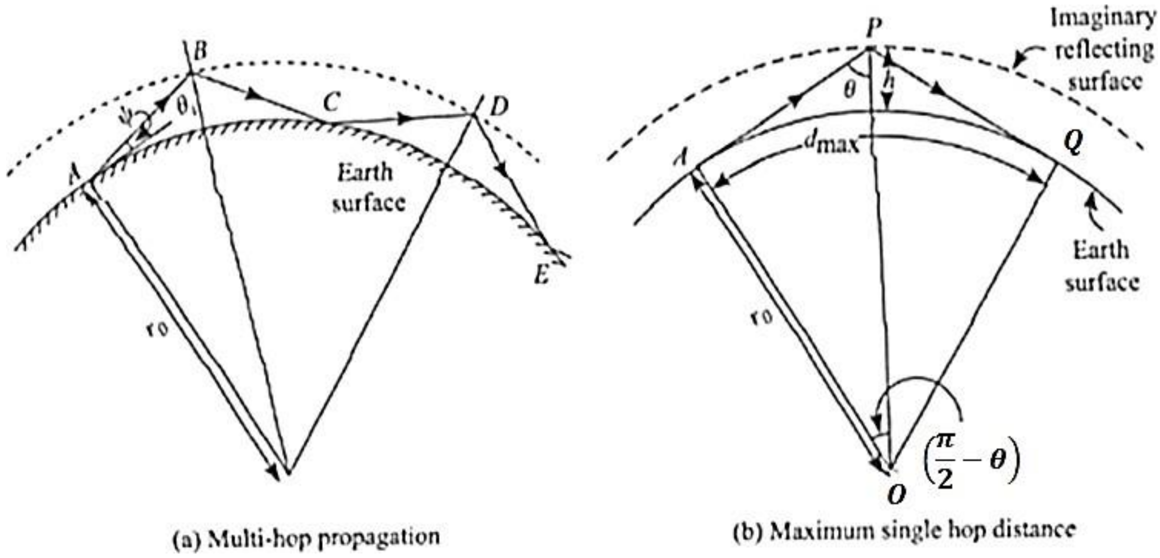


Fig. 5-17 Propagation of sky waves above the spherical earth

- The single-hop distance AC is a function of the grazing angle, ψ. The maximum value of the single hop distance occurs for ψ = 0 or horizontal launch as shown in Fig. 5.17 (b). The incidence angle at P is given by ;

$$\theta = \sin^{-1}\left(\frac{r_0}{r_0 + h}\right) \qquad \text{-------- (5.23)}$$

- Therefore the maximum single hop distance is

$$d_{max} = 2r_0\left(\frac{\pi}{2} - \theta\right) \qquad \text{-------- (5.24)}$$

- For example, the reflection from the E layer with $h = 100\ km$, the angle of incidence is

$$\theta = \sin^{-1}\left(\frac{6370}{6470}\right) = 79.91° = 1.395\ rad$$

- The maximum single-hop distance is

$$d_{max} = 2(6370)\left(\frac{\pi}{2} - 1.395\right) = 2240\ km$$

- Similarly, for the F layer, with a virtual height of 300 km, the maximum angle of incidence is 72.75° and the maximum single-hop distance is 3836 km.

FADING

- Fading is basically the undesirable variation in the intensity of the signal received at the receiver. Hence the fading is defined as the fluctuations in the received signal strength caused due to variations in height and density of the ionization in different layers.
- Basically the fading is the common characteristic of the high frequency short wave propagation i.e. sky wave propagation. At receiver, the strength of the signal received is the vector sum of the waves received.
- Because the waves leave from transmitter at same time but reach at the receiver through different paths. So the fading is caused due to interference between two waves of different path lengths.
- Various types of fading are as follows.

1. **Selective Fading** :
   - It is more dominant at high frequencies for which sky propagation is used.
   - The selective fading produces serious distortion of modulated signal.
   - Due to the selective fading, the amplitude modulated signals are seriously affected.
   - The AM signal are more distorted due to the selective fading rather than SSB signals. Hence to reduce the selective fading Exalted carrier reception and single side band system can be used.

2. **Interference Fading**:
   - As name indicates, it is the fading produced because of upper and lower rays of the sky wave interfering with each other. This is the most serious fading.
   - It is also produced due to the interference between a ground wave and a sky wave or between sky waves reaching receiver by different paths or different number of hops.
   - For a single sky wave frequency, interference fading takes place due to the fluctuations in the height of the ionospheric layer or due to the variation in the ionic

density of the layer.

- As ionosphere is subjected to the continuous small variations. Because of this, the length of the path that the reflected wave follows also undergoes small variations. Thus the relative phase of the wave reaching receiver varies randomly. Because of these conditions, the amplitude of the resultant also varies continously which is nothing but the interference fading.

 This can be minimized by using space diversity or frequency diversity reception.

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3. **Absorption Fading**:
   - This type of fading occurs due to the variations of single strength with the different amount of absorption of waves absorbed by the transmitting medium.
4. **Polarization Fading**:
   - When the sky wave reaches after the reflection, the state of polarization is constantly changing.
   - The polarization of the sky wave coming down changes because of the superposition of the ordinary and extra ordinary waves (which are having random amplitudes and phases) which are oppositely polarized.
   - Thus the polarization of the wave changes continuously with respect to antenna, which gives rise to the variations in the amplitude at the receiver. Such type of fading is called polarization fading.

5. **Skip Fading**:
   - At distances near the skip range or skip zone, the fading occurs which is called skip fading.
   - Due to the variations in the height and the density of the ionized layer, the point at which the wave can be received moves in or out of the skip zone.
   - Thus due to this the amplitude at the receiver also varies producing skip fading near the skip range.
   - The minimize the fading, the most common method is to use automatic volume control (AVC or AGC), in the receiver.

Note: Fading is the fluctuation in the received signal strength at the receiver or a random variation in the received signal. Can beminimized by Space diversity or frequency diversity.

SPACE WAVE PROPAGATION

- Space waves are useful in the frequency range of 30 MHz to 300 MHz. It is used in FM, TV and radar applications. In this propagation, wave propagates within the troposphere. It is the lowest portion of the atmosphere.
- Space wave consists of two components i.e. direct wave (line-of-sight, LOS) and indirect wave. Even though, both the wave namely direct wave and indirect wave are transmitted at the same time, with same phase, at the receiving end they may reach in phase or out of phase depending on the different path lengths.
- Thus at the receiving end, the signal strength is the vector addition of the strengths of the direct and indirect waves. When the two waves are in phase, the strength of the signal at the

receiver will be stronger. Similarly if the two waves are out of phase, the strength of the signal at the receiver will be weaker.

- The space wave propagation is mainly used in VHF (Very High Frequency) band as both previous modes namely ground wave propagation and sky wave propagation both fail at very high frequencies.

TROPOSPHERIC PROPAGATION:

- The tropospheric region extends from the surface of the earth to a height of about 10 km at the poles and 18 km at the equator. The temperature of this region decreases with height at the rate of about 6.5°C per km and falls a minimum value about −52°C at its upper boundary.
- In this region, the cloudes are formed. Next to the troposphere, stratosphere exists. The propgation through the troposphere takes place due to mechanisms such as diffraction, normal refraction, abnormal reflection and refraction and tropospheric scattering.
- In troposphere, slight bending of radio waves occurs and causes signals to return to earth beyond the geometric horizon. Troposphere bending is evident over a wide range of frequencies, although it is most useful in the VHF and UHF regions. Radio signals can be trapped in the troposphere, travelling a longer distance than normal before coming back to the earth surface.
- Instead of gradual changes in the atmospheric conditions, sometimes distinct regions are formed and regions that have significantly different densities try to bend radio waves passing between regions. However, in a non-homogeneous atmosphere whose index of refraction decreases with height, rays of sufficiently small initial elevation angle are refracted downward with a curvature proportional to the rate of decrease of the index of refraction with height.
- Out of different mechanism of troposphere wave propagation diffraction, abnormal reflection and refraction, and troposphere scattering, the normal refraction is the main mechanism for most of troposphere propagation phenomenon.
- The dielectric constant (hence refractive index) of the atmosphere which varies above the earth and set mostly by the moisture contains is a primary factor in the troposphere refraction. When the wave passes between mediums of different densities, its path bends by an amount proportional to the difference in densities. Especially, at UHF and microwaves two cases of tropospheric propagation are observed.

⇨ **Index of refraction:**

- In troposphere, the relative dielectric constant is slightly higher than unity due to the prescence of the atmosphere and in particular water vapour. The relative dielectric constant is a function of the temperature, pressure and humidity (water vapour). The typical value of ϵ_r at the surface of the earth is found to be 1.00579.
- The value decreases as a function of height above the surface of the earth. We know that the velocity of the electromagnetic wave in a medium depends on the dielectric constant of the medium.

$$v = \frac{c}{n} = \frac{c}{\sqrt{\epsilon_r}} \qquad \text{-------- (5.25)}$$

where c is the velocity in vaccum and $n = \sqrt{\epsilon_r}$ is the refractive index of the medium.

- At the surface of the earth (mean sea level), the refractive index of air is 1.000289. Therefore, it is common practice to work with a parameter known as the refractivity N. The refractivity is related to refractive index by the following equation.

$$N = (n - 1) \times 10^6 \qquad \text{-------- (5.26)}$$

TROPOSCATTER PROPAGATION:

- Troposcatter is a mechanism by which propagation is possible by the scatter and diffracted rays. The scattering takes place in the tropospheric region. This mode of propagation occurs in VHF, UHF and microwave band.
- UHF and microwaves signals were found to be propagated much beyond the line of sight propagation through the forward scattering in the tropospheric irregularities.
- This mechanism helps to get unexpectedly large field strengths at the receivers even when they are is shadow zone. It is possible to achieve a very reliable communication over a range of 160 km ato 1600 km by using high power transmitter and high gain antennas.
- The tropospheric scattering phenomenon can be used to establish a communication link over a distance much beyond the radio horizon. The troposphere can scatter electromagnetic waves due to its inhomogenous nature. The tropospheric scattering has been attributed to the blobs of refractive index changes and turbulence. These could be due to sudden changes in the temperature or humidity or the presence of dust particles.
- Waves passing through such turbulent regions get scattered. When $\boldsymbol{\lambda}$ is large compared to the size of the turbulent eddies, waves scatter in all the directions. When $\boldsymbol{\lambda}$ is small compared to these irregularities then most of the scattering takes place within a narrow cone surrounding the forward direction of propagation of the incident radiation.

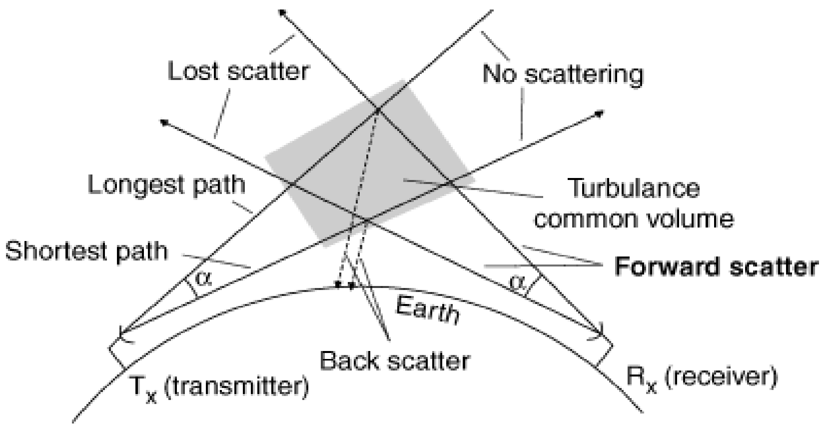

Fig. 5-18 Illustration of a troposcatter link.

- When the wavelength is small (frequency is high) than the eddies, forward scattering dominates into the cone of angle $\boldsymbol{\alpha}$. The angle $\boldsymbol{\alpha}$ should be very small.
- To receive scattered signal at a point well beyond the horizon, the transmitting and receiving antennas must be of high gain and must be so oriented that their beams overlap in

a region where forward scattering is taking place. The scattering angle should also be as small as possible. This process is shown in Fig. 5-18. Since the scattering process is of random nature, the scattered signals continuously fluctuate in amplitude and phase over a wide range.

- Troposcatter can be used to establish communication links in the UHF and microwave frequency bands. These links typically have a range of up to a thousand kilometers and can have bandwidths of a few MHz. Troposcatter links can be used in multi-channel telephony and television applications.

Features of troposcatter propagation:

- It is useful for propagation in the range of 100 MHz to 10 GHz.
- It produces undesirable noise and fading which may be minimized to certain extent by diversity reception.
- The field strength received is usually on the order of $d^{1/7}$ or $d^{1/8}$ where d is the distance between the transmitter and receiver.
- Since the signal strength is very weak, high gain antennas are required for reception.
- The propagation exhibits seasonal variation.
- The forward scatter propagation is useful for point to point communications and radio or television relay links.

DUCT PROPAGATION:

- The VHF, UHF and microwave frequencies are the frequencies which are neither propagated along the surface of the earth nor reflected by ionosphere. But in the troposphere region, the high frequency waves are refracted and transmission takes far beyond line-of-sight (LOS) distance.
- An atmosphere where the dielectric constant is assumed to decrease uniformly with height to value equal to unity at which air density is supposed to be zero is commonly called normal atmosphere or standard atmosphere.
- There are different air regions or layers one above other with different temperatures and water vapour contents. In one of the regions, there is a region where dN/dh is negative. In this region, the curvature along which the radio waves pass is slightly greater than that of the earth.
- Due to this, the wave originally directed almost parallel to the surface of the earth gets trapped in such regions. The energy originating in this region propagates around curved surfaces in the form of series of hops with successive reflections from the earth as shown in the Fig. 5-19. This phenomenon is called super refraction or duct propagation. Two boundaries of surfaces between two air layers form a duct which guide the radio waves between walls i.e. boundries.
- The concepts like line of sight and diffraction cannot be applied when the wave propagates through duct and it is found that the energy travels high distances round the earth without much attenuation.

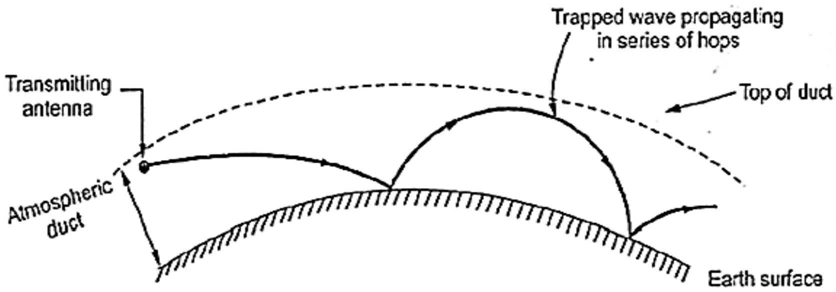

Fig. 5-19 Duct propagation

- The concept of wave trapping can be considered as a phenomenon similar to wave guide. But the main difference between waveguide and duct propagation is that in wave guide all the modes are confined within guide only. But in case of duct propagation, part of energy within duct may escape to the space as shown in the Fig. 5-20. There is a limit on the wavelength of the signal of maximum value λ_{max} to be trapped in duct. It is the maximum wavelength for which the duct propagation holds good. If the wavelength of the signal exceeds the value λ_{max}, then duct effect vanishes almost completely. The value of λ_{max} is given by,

$$\lambda_{max} = 2.5\ h_d \sqrt{\Delta N} \times 10^{-6} \qquad \text{-------- (5.27)}$$

where ΔN = change in N value across height of duct

h_d = height of duct

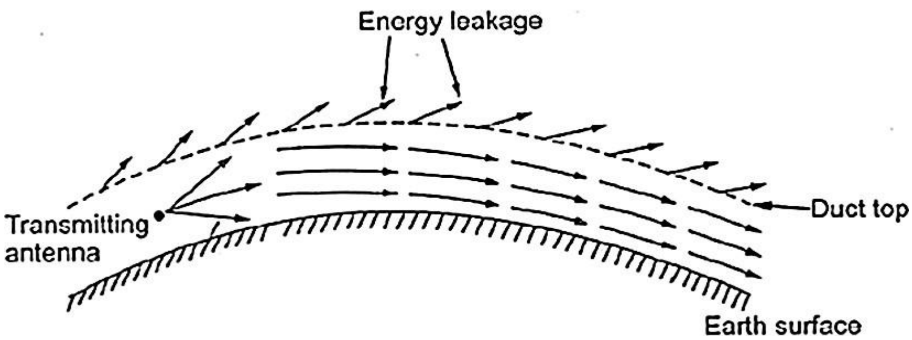

Fig. 5-20 Duct propagation as leaky wave guide

- In general, the duct height h_d ranges from 10 to hundreds of meters. While the ΔN value is typically 50 units. So considering these values, the phenomenon of duct propagation is found mostly in UHF (ultra high frequency) and microwave frequency regions.
- Moreover the duct propagation is possible only if height of transmitting antenna is less than that of duct height. If the transmitting antenna exists considerably above duct, there is comparatively less effect of presence of duct on the signal either inside or above duct.

Post – Test MCQs:

1. The portion of the atmosphere which extends up to 20 Km from the earth surface is called
   a. Traposphere
   b. Statosphere
   c. Inosphere
   d. none of the above

Ans: a

2. The ionized layer that extends from about 50 km above the surface of the earth to several thousand kilometers is known as.

   a. Troposphere
   b. Ground wave
   c. Surface wave
   d. ionosphere

Ans: d

3. The ground waves are produced in ____________ which are placed very close to surface of the earth speed of sound
   a. An Antenna
   b. Vertical polarized antenna
   c. Horizontal polarized antenna
   d. Vertical antenna

Ans: b

4. the frequency which can be reflected back to earth for some specific angle of incidence
   a. TUF
   b. MUF
   c. Gyro Frequency
   d. All the above

Ans:b

5. The following one is type of fading
   a. Abnormal Fading
   b. Selective and Absorption fading
   c. Normal Fading
   d. All the above

Ans: b

6. the frequency normally used for the ionospheric propagation is known as
   a. optimum working frequency
   b. Maximum usable frequency
   c. Duct propagation
   d. Optimum frequency

Ans: a

7. The Characteristics of E layer is
    a. It exists next to D layer at an average height of 100 km.
    b. Its thickness is about 25 km.
    c. Its critical frequency is 4 MHz.
    d. All the above

Ans:d

8. The Characteristics of F1 layer is
    a. It exists at a height of about 180 km in day-time.
    b. Its thickness is about 20 km.
    c. Its critical frequency is 5 MHz.
    d. All the above.

Ans: d

9. The critical frequency is defined as
    a. *the lowest frequency that can be reflected back to the earth by a particular layer for a vertical incidence. It is denoted by* fcr
    b. *the highest frequency that can be reflected back to the earth by a particular layer for a vertical incidence. It is denoted by* fcr
    c. *the highest frequency that can be reflected back to the earth by a particular layer for a horizontal incidence. It is denoted by* fcr
    d. *the lowest frequency that can be reflected back to the earth by a particular layer for a horizontal incidence. It is denoted by* fcr

Ans: b

10. In selective fading the following singal is seriously affected
    a. Amplitude modulated signal
    b. Freauency modulated signal
    c. Phase modulated signal
    d. None of the above

Ans: a

11. The F1-Layer critical frequency is
    A) 1Mhz
    B) 10Mhz
    C) 3Mhz
    D) 5Mhz

Ans: d

12. Waves that arrive at the receiver after reflection in the ionosphere is called ________
    a. Sky Wave
    b. Ground Wave
    c. Scattering wave
    d. none of the above

Ans: a

13. The critical frequency of the D layer is
    a. 5Khz
    b. 100Khz
    c. 250 Hz
    d. 500 Mhz

Ans: b

14. The height at a point above the surface at which the wave bends down to the earth is called
    a. actual height
    b. true height.
    c. both a and b
    d. None of the above

Ans: c

14. MUF is usually _______ times of critical frequency.
    a. 2 to 4
    b. 5 to 6
    c. 6 to 7
    d. 3 to 4

Ans: d

Conclusion:

At the end of the topic, students will be able to

- understand the basic concept of various types of propagation
- get exposure to sky wave propagation characteristics

References:

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2. Edward C. Jordan and Keith G. Balmain” Electromagnetic Waves and Radiating Systems” Prentice Hall of India, 2nd Edition 2011.
3. R.E. Collin,” Antennas and Radio wave Propagation”, McGraw Hill 1985.
4. Constantine.A. Balanis “Antenna Theory Analysis and Design”, Wiley Student Edition, 4th Edition 2016.
5. Rajeswari Chatterjee, “Antenna Theory and Practice” Revised Second Edition New Age International Publishers, 2006.
6. S. Drabowitch, “Modern Antennas” Second Edition, Springer Publications, 2007
7. Robert S. Elliott “Antenna Theory and Design” Wiley Student Edition, 2006.

Assignments:

1. Explain about the Structure of Atmosphere with neat diagram
2. Write short notes on Ground wave propagation and Troposcatter propagation
3. Explain the following. A. Skip distance B. Virtual Height C. MUF D. Critical Frequency
4. Explain about Duct propagation and Multi hop propagation